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Estimating the Eutectic Composition of Simple Binary Alloy System Using Linear Geometry

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Abstract

A simple linear equation was developed and applied to a hypothetical binary equilibrium diagram to evaluate the eutectic composition of the binary alloy system. Solution of the equations revealed that the eutectic composition of the case study Pb – Sn, Bi – Cd and Al – Si alloys are 39.89% Pb, 60.11% Sn, 58.01% Bi, 41.99% Cd and 90.94% Al, 9.06% Si respectively. These values are very close to experimental values. The percent deviation of analytical values from experimental values ranged between 2.87 and 5% for the three binary systems considered, except for Si – Al alloy in which the percent deviation for the silicon element was 22%.

It is concluded that equation of straight line could be used to predict the eutectic composition of simple binary alloys within tolerable experimental deviation range of 2.5%.

Keywords

Estimating; Eutectic Composition; Binary Alloy System; Linear geometry.

Introduction

Alloys are developed from combination of two or more metallic and/or non-metallic element [1]. The primary objective of alloying is to improve mechanical properties, though reduction in some other properties such as conductivity and corrosion resistance do accompany the process. Alloy systems are designated by the number of elemental component that made up the system. Thus, a 2-element component alloy system is referred to as binary alloy system and a 3-element component system is called ternary alloy system. The characteristic and thermal behaviour of the alloy system are captured in the phase diagram. The phase diagram describe the relationship between composition and temperature in determining the phase fields of the various constituents of the diagram. The phase diagram is constructed from the temperature – time (cooling curve) history of each of the constituents [2]. The cooling curve of systems is evaluated by measuring the rate of thermal gradient between the start and end of solidification for each individual component of the cooling curve system [3]. This is in turn transferred to the temperature-composition axes to construct the phase diagram.

For instance, the binary phase diagram of two components, A and B (Figure 1) depicts the temperature of the system as a function of percent composition of the component.

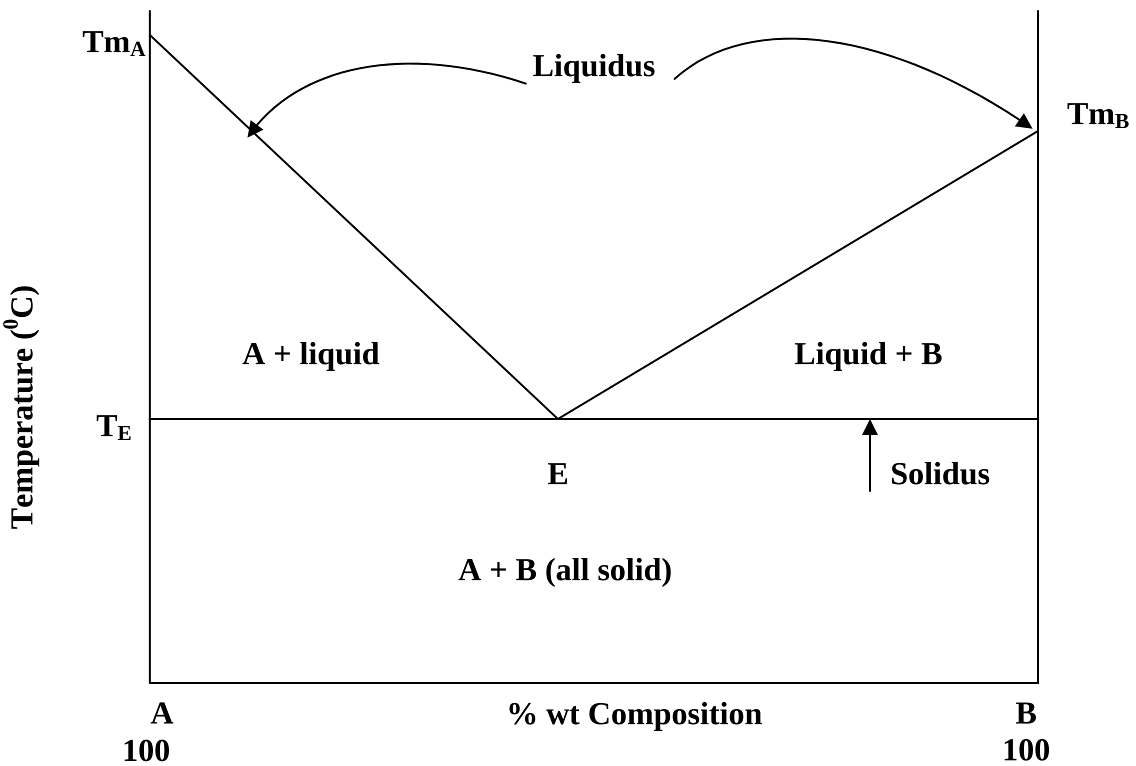

Figure 1. Binary Phase diagram of two-components [4]

When components A and B are mixed in varying proportion (0-100%), and heated until all the components are completely melted, the melting point is found to be lowered compared to the melting temperature of the pure components [4]. The plotting of the temperature against the corresponding composition of the component mixture outlined the trajectory Tm_2ET_{mB} known as the liquidus curve. The lowest melting point E of the curve defines the eutectic temperature [T_E] and the corresponding composition is called the eutectic composition or alternatively, eutectic point for both temperature and composition. The point is called eutectoid point when the transformation is from homogenous solid solution to two different solids. This is common in iron carbon equilibrium diagram [5]. At this point, components A and B melt uniformly at the same temperature and the liquid phase co-exist with the solid phase of A and B. The eutectic point is equally available in phase diagrams of many other alloy systems.

The eutectic point has much significance in metallurgy and materials science. It is the most desirable point in foundry/casting as it ensures that casting is accomplished at minimum temperature. This help to reduce energy cost. The eutectic point equally impact optimum properties in alloy system [6].

Before now, most efforts at determining the eutectic point had been experimental based. This is done through any of differential scanning calometry [7,8], metallographic technique [9,10] and x-ray diffraction techniques [11]. Each of these systems shares their unique limitations. Besides, they are cumbersome and expensive.

In the present work, a new analytical method is explored to estimate the eutectic composition of simple binary alloy system. The analytical model is validated against some known experimental eutectic compositions. The percent variation is less than five percent. The central element in the developed analytical model is that the eutectic temperature must be known in order to be able to estimate the eutectic composition using linear geometry.

Basis of the Analysis

The mixture of two or more substance that melt at the lowest possible temperature is called the eutectic mixture. The eutectic mixture is approximated by two metals such as lead and tin, bismuth and cadmium, aluminum-silicon. The lead and tin binary system is reproduced in Figure 2.

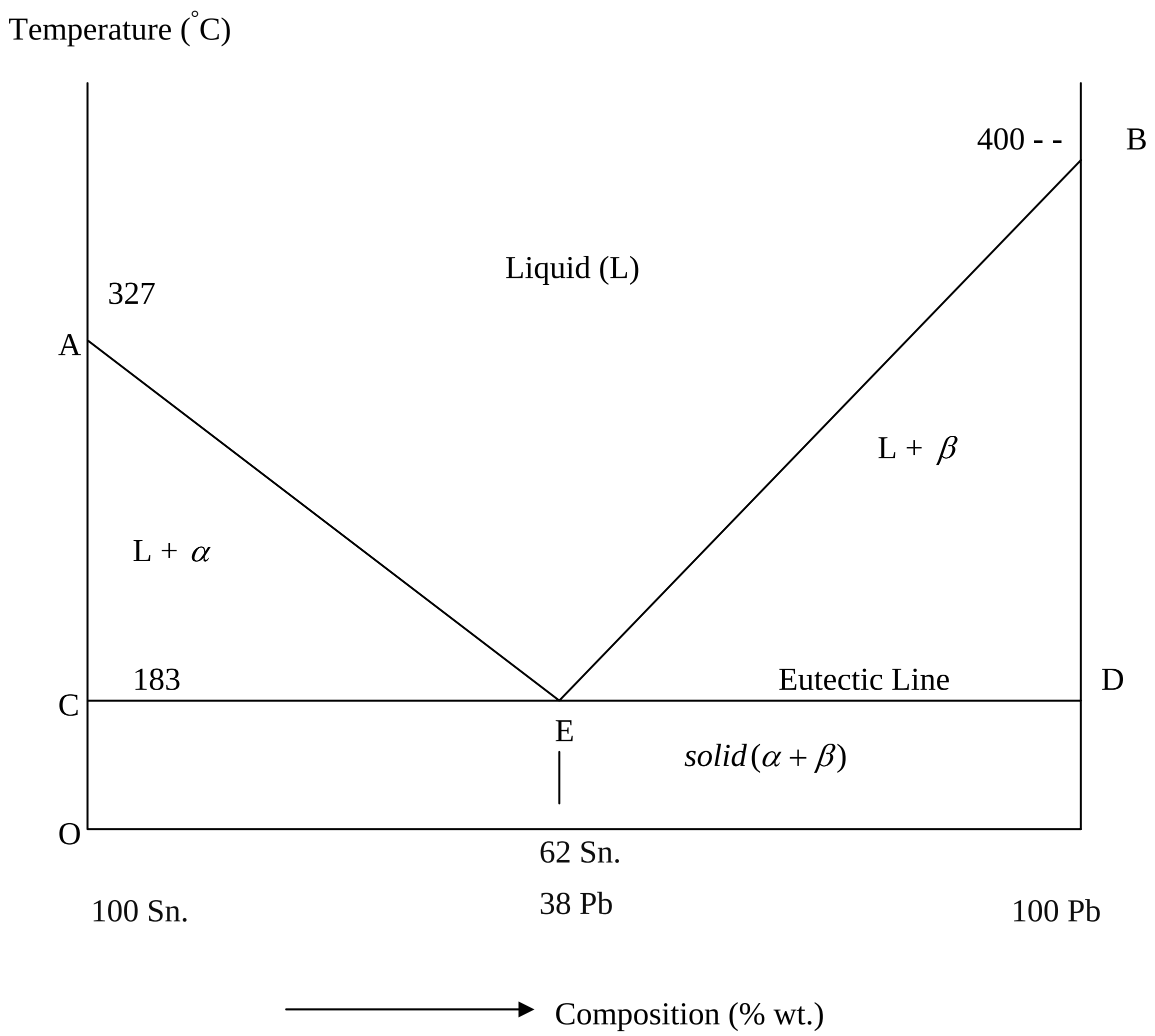

Figure 2. Equilibrium diagram of Sn. -Pb. alloy system

The freezing point of lead is 400°C while that of tin 327°C. Figure 2 revealed that adding increasing amount of lead to tin progressively depresses the freezing point of tin along the line ZE. Similarly, the freezing point of lead progressively depresses along the line WE when increasing amount of tin is added to lead. These depressions of freezing point lines meet at a point called the eutectic point. The eutectic point for a tin-lead binary system is 38% wt Pb, 62%wt Sn at 183°C [1].

Issue 12, January-June 2008
p. 232-242

The Development of the Model

The tin-lead alloy system is deployed in this analysis. The liquidus lines start from the melting point of the two metals and terminate at an assumed eutectic point E.

In the developed model, the eutectic temperature is taken as the reference temperature and is assigned zero on the temperature scale. The basis for the assumption is that representing the eutectic temperature (183°C) as 0°C and subtracting it from the melting points of lead and tin to get lower values will not affect the result that is generated. The essence of the scaling down is to get an origin for the reference Y-axis used for the analysis (Figure 3).

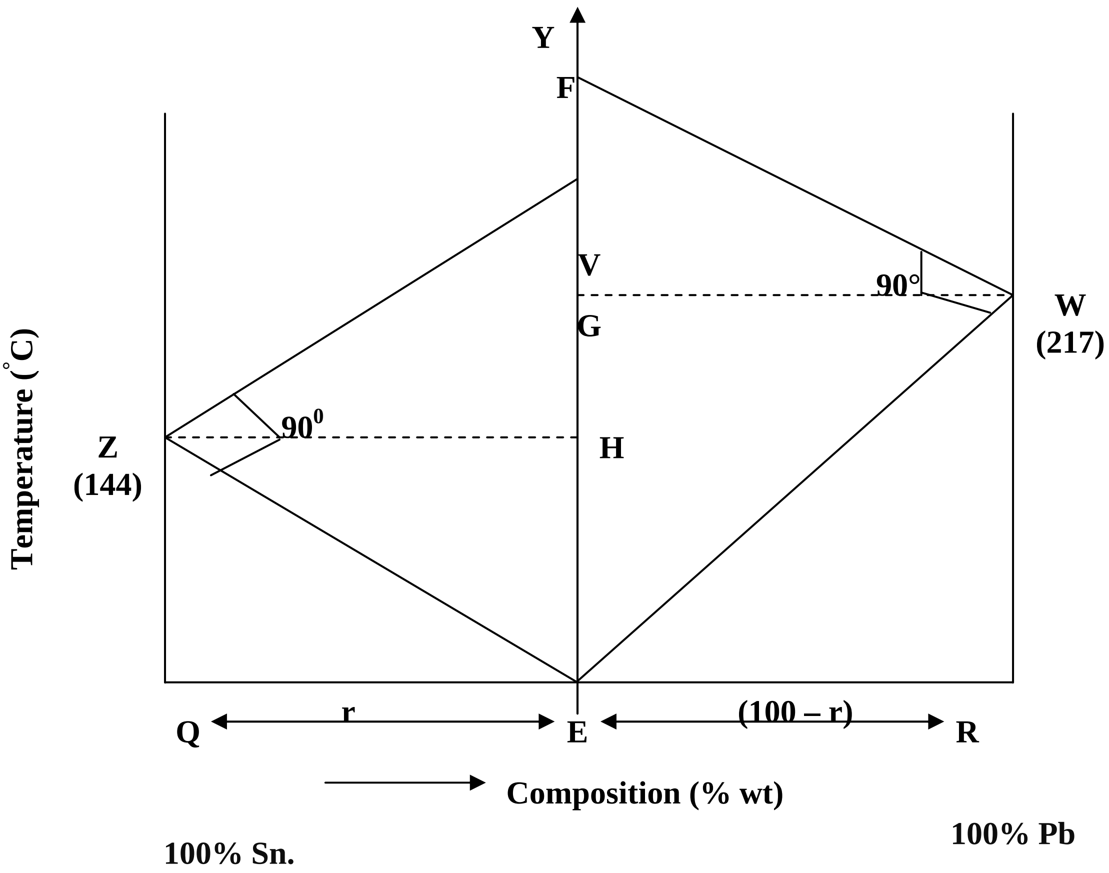

Figure 3. Scaled down diagram of the Sn-Pb alloy system showing the reference Y-axis with the intercepting straight lines used for the analysis

Consequently, W (melting point of lead) is represented by the value 400 – 183 = 217°C and Z (melting point of tin) by the value of 327-183 = 144°C. The two liquidus lines were joined from points 217°C and 144°C respectively to meet at the assumed eutectic point E.

A reference Y-axis line EY was drawn from E.

A line WF perpendicular to WE was drawn from W to cut EY at F. A similar line ZV perpendicular to ZE cut EY at V.

The scales of the composition and temperature axis are 1cm represent 1 unit of either composition or temperature.

Therefore from Figure 3; If QE = r cm; ER= (100 – r) cm, the gradient of

$$WE = m = \frac{217}{100 - r} \quad (1)$$

$$ZE = n = \frac{144}{r} \quad (2)$$

Relating eqn. (1) and (2) gives

$$\frac{n}{m} = \frac{144}{r} \times \frac{100 - r}{217} = \frac{14400 - 144r}{217r} \Rightarrow n = \frac{m(14400 - 144r)}{217r} \quad (3)$$

The equation of a straight line is generally given as

Y = Px + C

where P = gradient of the line; C = its intercept on Y-axis; Y = dependent variable; X = independent variable.

If the gradient of a straight line is P; then the gradient of another straight line perpendicular to it is given as -1/P.

Therefore, the equation of line WE is given as

$$\begin{aligned} Y &= mx + 0 \text{ (since the intercept of WE on the Y-axis is 0)} \\ Y &= mx \end{aligned} \quad (4)$$

The equation of the perpendicular line WF is given as

$$Y = \frac{-x}{m} + 217 + K \quad (5)$$

where K = FG.

Issue 12, January-June 2008
p. 232-242

Similar analysis gives the equation of line ZE as

$$Y = \frac{m(14400 - 144r)x}{217r} + 0$$

Intercept of ZE on the Y-axis = 0

$$Y = \frac{m(14400 - 144r)x}{217r} \quad (6)$$

The equation of perpendicular line ZV is given as

$$Y = \frac{-(217r)x}{m(14400 - 144r)} + 144 + J \quad (7)$$

where J = VH.

Subtracting eqn. (7) from eqn. (5)

$$0 = \frac{-x}{m} + \frac{(217r)x}{m(14400 - 144r)} + 73 + K - J \quad (8)$$

Subtracting eqn. (6) from eqn. (5)

$$0 = \frac{-x}{m} - \frac{m(14400 - 144r)x}{217r} + 217 + K \quad (9)$$

Subtracting eqn. (9) from eqn. (8)

$$0 = \frac{217r)x}{m(14400 - 144r)} + \frac{m(14400 - 144r)x}{217r} - 144 - J \quad (10)$$

Subtracting eqn. (7) from eqn. (4)

$$0 = mx + \frac{(217r)x}{m(14400 - 144r)} - 144 - J \quad (11)$$

Subtracting eqn. (11) from eqn. (10) gives

$$0 = m\frac{14400 - 144r)x}{217r} - mx \quad (12)$$

Dividing eqn. (12) by *mx* gives

$$0 = \frac{(14400 - 144r)}{217} - 1 \tag{13}$$

Rearranging eqn. (13) gives

$$14400 - 144r - 217r = 0$$
$$316r = 14400 \tag{14}$$
$$r = 14400/316 = 39.89\ (cm) = QE$$

Since 1cm represent 1 unit of composition; 39.89 cm $\equiv$ 39.89%

Relating this to the lever rule implies that Pb eutectic composition is 39.89%. Thus, the eutectic composition of Tin (Sn) is (100 – 39.89) % = 60.11%.

Therefore analytical eutectic composition for tin-lead binary system (E_C) is 39.89% Pb, 60.11% Sn.

However, the experimental eutectic composition for tin-lead system are 38% Pb; 62% Sn [1].

The method was applied to other eutectic alloy systems. These are the bismuth-cadmium and aluminum-silicon alloys systems.

For the bismuth-cadmium alloy system, the freezing points of bismuth and cadmium are 271°C and 321°C respectively. The eutectic temperature is 140°C. The experimental eutectic composition is 60% Bi, 40% Cd.

For the aluminum-silicon alloy system, the freezing points of aluminium and silicon are 660°C and 1410°C respectively [12]. The experimental eutectic composition of the Al-Si alloy system is 11.6% Si, 88.4% Al [1].

The governing equation developed in the present work for the calculation of the eutectic composition of an alloy by analytical method is given in equation (15).

$$14400 - 144r - 217r \tag{15}$$

This can be generally written as

$$A(100) - Ar - Br = 0 \tag{16}$$

where: r = composition of the metal that has the higher freezing point; A = thermal gradient of the metal that has the lower freezing (freezing point minus eutectic temperature); B = thermal gradient of the metal that has the higher freezing (freezing point minus eutectic temperature)

Therefore for Bi – Cd system (Figure 4)

Issue 12, January-June 2008
p. 232-242

- A = (271-140) = 131; A(100) = 13100
- B = (321-140 = 181

Therefore substituting for the various values of A & B in eqn. (16) gives

$13100 - 131r - 181r \Rightarrow r = 41.99\%$

Thus, eutectic composition of cadmium, Cd = 41.99%

Therefore, Bismuth is given as 100 – 41.99 = 58.01%

For the aluminum – silicon alloy system

- A = (660-577) = 83; A(100) = 8300
- B = 1410-577 = 833

Substituting these values in the general eqn. (16), gives

8300 – 83r – 833r = 0

r = 9.06%

The eutectic composition of silicon = 9.06% Wt Si

Therefore aluminum = (100-9.06%) wt Al = 90.94% wt Al

Therefore, eutectic composition to Al – Si alloy by analytical method using linear geometry is 9.06% wt Si, 90.94% wt Al.

The analysis of the result relative to experimental values is tabulated in Table 1.

Table 1. Differential Analysis of Experiment and Analytical Methods of Determining Eutectic Composition of Simple Binary Alloy Systems

Alloy System	Experimental Values	Analytical Values	% Variation
Sn – Pb	62% Sn 38% Pb	60.11% Sn 39.89% Pb	Sn = 3% Pb = 5%
Bi – Cd	60% Bi 40% Cd	58.01% Bi 41.99% Cd	Bi = 3.3% Cd = 5%
Si – Al	11.6% Si 88.4% Al	9.06% Si 90.94% Al	Si = 21.89% Al = 2.87%

The analysis of Table 1 revealed that the percent deviation of analytical value from experimental value ranged between 2.87 and 5% for the three systems considered, except for the Si-Al alloy where the percent deviation of the silicon composition was rather high at 22%. This apparent exceptional high percent deviation from experimental value might be due to the partial solubility of silicon element in aluminum.

Attempt was made to extend the work to iron-carbon phase diagrams. The method has not been successful yet owing to the various polymorphic transformations that exist in iron-carbon system in the temperature range 910°C and 1400°C.

Conclusion

A simple linear general equation of the form A (100) – Ar + Br = 0 has been developed and applied to analytically estimate the eutectic composition of simple binary alloy system. The solution of the equation revealed that the eutectic composition of the case study Pb-Sn, Bi-Cd and Al-Si alloy systems are 39.89%Pb, 60.11%Sn, 58.01% Bi, 41.99% Cd and 90.94% Al 9.06% Si respectively. These values are very close to experimental values. The percent deviation of analytical value from experimental value ranged between 2.87 and 5% for the three binary systems considered except for Si-Al alloy in which the percent deviation for the silicon element was 22% owing probably to the partial solubility of silicon in aluminum.

It is concluded that eutectic composition of simple binary system could be analytically determined by using linear geometry. This probably is a novel approach to removing the difficulty encountered in the conventional differential thermal analysis, metallographic and crystallographic techniques.

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