Optimum Pipe Size Selection for Turbulent Flow

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Abstract

Pipelines are normally designed to deliver fluid at the required head and flow rate in a cost effective manner. Increase in conduit diameter leads to increase in annual capital costs, and decrease in operating costs. Selection of an optimum conduit diameter for a particular fluid flow will therefore be a vital economic decision. This paper presents a computer aided optimisation technique for determination of optimum pipe diameter for a number of idealized turbulent flow. Relationships were formulated connecting theories of turbulent fluid flow with pipeline costing. These were developed into a computer program, written in Microsoft Visual C++ language, for a high-level precision estimate of the optimum pipe diameter, through the least total cost approach. The validity of the program was ascertained through case studies, representative of fluids with different densities and compressibility. The optimum conduit diameter was found to increase linearly with increase in compressibility.

Keywords

Pipe diameter, Turbulent flow, Optimisation, Costs, density

Introduction

Pipe flow, as used in this work, refers to flow in a circular closed conduit entirely filled with fluid, which an economic investigation indicates are sufficiently large and continuous to justify the investment. Pipelines are one of the lowest - cost means of transportation [1], with notable applications in oil and gas conveyances as well as water distribution systems [2].

The head losses in piping installations include the energy or head required to overcome resistance of the pipeline and fitting in the pumping system. Friction exists on both the discharge and suction sides of a pump and energy loss in pipe flow depends on the fluid velocity, density, viscosity, and conduit dimension [3].A number of equations governing fluid flow in pipelines have been developed by authors [4,5]. The most widely used ones include the Scobey, Darcy-Weisbach and Hazen-Williams formula.

In selecting pipe size for different applications, small pipe may require a lower initial investment but the head loss due to friction is greater and this increases the energy cost. A larger pipe will save more in energy cost than the additional investment. Besides, the larger pipe may so reduce the total pump head that a higher and lower priced pump and power unit may be used [6]. Hence, an optimum pipe diameter must exist. This optimum value can be determined by combining the principles of fluid dynamics with cost analysis. A similar approach has been used by authors [4, 7].

This work is aimed at determining the optimum pipe diameter for turbulent fluid flow with different compressibility through integration of cost analysis into the principles of fluid dynamics, and adopting the least total cost approach through computer simulation in Microsoft Visual C++.

Material and Method

For flow of fluid through a conduit of constant diameter, the fixed or annual pumping cost can be determined using; (7)

$$C_1 = \frac{P_d C_e Q t}{p\eta} \tag{1}$$

where:

C_1 = the annual pumping cost

P_d = the frictional pressure drop, (kNm^{-2})

Q = fluid flow rate (kgs^{-1})

C_e = cost of electrical energy, (N kWhr^{-1})

t = operational hours per year; (hr. yr^{-1})

ρ = fluid density, (kgm^{-3})

η = Efficiency of motor and pump, (%)

Fluid compressibility can be expressed in terms of fluid density as:

$$\gamma = \frac{1}{\rho} \tag{2}$$

where;

γ =compressibility, (m^{3}kg^{-1})

In pipe flow problems, the friction factor is a function of Reynold's number and characterizes the nature of flow. For high Reynold's number; (Re > 2000), the flow is said to be turbulent and the friction factor is given as: [8]

$$f = \frac{0.04}{\mathrm{Re}^{0.16}} \tag{3}$$

where:

Re = Reynold's number,

f= fanning friction factor.

Using equation (2), the Reynold's number can be written in the form:

$$\mathrm{Re} = \frac{4Q}{\gamma\mu\pi d_i} \tag{4}$$

where: μ = fluid viscosity, (mNm^{-2}s)

d_i = inside diameter of pipe, (mm)

Authors [2, 9] have integrated equations (3) and Reynold's number expression, in terms of fluid density, into the fanning pressure drop equation, to obtain:

$$P_d = 4.13\times10^{10} Q^{1.84} \mu^{0.16} \rho^{-1} d^{-4.84} \tag{5}$$

Using (1), (2), (4) and (5), the annual pumping cost takes the form:

$$C_1 = 4.13\times10^{10} Q^{2.84} \mu^{0.16} \gamma^2 d^{-4.84} t C_e \tag{6}$$

The operating cost or piping cost has been expressed in the form: (4)

$$C_2 = C_p d^n (1+F)(a+b) \tag{7}$$

where:

C_2 = operating cost ,

C_p = cost per unit length of pipe, (Nmm^{-1})

F = ratio of total costs for fittings and installation to purchase cost for new pipe

a = capital charge, (%)

b = maintenance charge (%) and,

n is dependent on the current cost of piping. (2)

For steel pipes, n ≅ 1.5 for d ≥ 25.0mm while n ≅ 1.0 for d ≤ 25.0mm. The value of F generally ranges from 1.5 to 6.75. (4)

The total annual cost, C_T , is the addition of the fixed cost and the piping cost, i.e.:

$$C_T = C_1 + C_2 \quad (8)$$

Combining equations (1), (7) and (8) the total annual operational cost becomes:

$$C_T = C_p d^n (1+F)(a+b) + 4.13\times 10^{10} Q^{2.84} C_e T \mu^{0.16} \gamma^2 d^{-4.84} \quad (9)$$

A computational iterative technique is employed in the solution of the problem. This method has been employed by authors (9,10) for the solution of related optimization problems.

With an initial guess value of the internal diameter of pipe, known values for other parameters in equations (4) and (9), the nature of flow is determined using equation (4) and the total annual cost is evaluated using equation (9). This procedure is repeated with increase in the value of the internal diameter such that:

$$d(i+1) = d_i + \Delta d \quad (10)$$

with the total cost determined in each case until the total cost function passes through a minimum point. The internal diameter corresponding to the least total cost is taken as the optimum. Using the above procedure, for different hypothetical values of compressibility, and, keeping all other parameters constant, the optimum pipe diameter was determined in each case.

For quick and high level precision results, a computer program, written in Microsoft Visual C++ 6.0 is developed. The program, structured in user-screen interactive form was tested using typical input data.

Review of the Program

The program prompts and accepts input data such as filename, fluid flow rate, fluid compressibility, fluid viscosity, cost of electrical energy, hours of operation per year, the initial value of pipe diameter efficiency of compressor and pump, small increment in pipe diameter, value of maintenance charge, total cost of valves, fittings and erection, purchase cost for new one millimetre diameter per millimetre of pipe length and value of capital charge. The program then determines the nature of flow through eq. (4) and computes the total annual cost through eq. (9). The computation is carried out via 55 iterations and the results are generated in a user-defined file.

Results

Data from the case - study presented in Table 1 were fed into the program for validation. The results obtained are presented in Table 2. Graphical illustration of the result is also shown in Fig 1.

Program Validation

The computer code was validated using the case study presented in Table 1.

Table 1. Input data for Program Validation

S /N	Parameters	Value
1	Compressibility	$1.136363 \times 10^{-3} m^3kg^{-1}$
2	Operating time	$8000\ hrs.yr^{-1}$
3	Mass flow rate	$10\ kgs^{-1}$
4	Viscosity	$1.1 \times 10^{-3}\ Nsm^{-2}$
5	Elect. Energy cost	$N2.50kwhr^{-1}$
6	Cost / mm length of pipe	$N5.0mm^{-1}$
7	Capital charge	10%
8	Maintenance charge	5%
9	Ratio of total costs for fittings and installation to new pipe purchase cost	2.25
10	Efficiency	0.6
11	Initial guess for internal pipe diameter	31.9mm
12	Increment in pipe diameter	0.05mm

Table 2. Output of the Program for the Case Study

Inside Diameter of pipe	Total annual cost
31.95	65700.9
32.00	65639.7
32.05	65580.0
32.10	65522.0
32.15	65465.4
32.20	65410.4
32.25	65357.0
32.30	65305.0
32.35	65254.5
32.40	65205.4
32.45	65157.9
32.50	65111.7
32.55	65067.0
32.60	65023.7
32.65	64981.8
32.70	64941.3
32.75	64902.2
32.80	64864.4
32.85	64827.9
32.90	64792.8
32.95	64759.0
33.00	64726.6
33.05	64695.4
33.10	64665.4
33.15	64636.8
33.20	64609.4
33.25	64583.2
33.30	64558.3
33.35	64534.6
33.40	64512.1
33.45	64490.8
33.50	64470.7
33.55	64451.7
33.60	64433.9
33.65	64417.3
33.70	64401.8
33.75	64387.4
33.80	64374.2
33.85	64362.0
33.90	64351.0
33.95	64341.0
34.00	64332.1
34.05	64324.3
34.10	64317.6

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34.15	64311.8
34.20	64307.2
34.25	64303.5
34.30	64300.9
34.35	64299.3
34.40	64298.6
34.45	64299.0
34.50	64300.3
34.55	64302.7
34.60	64305.9
34.65	64310.2

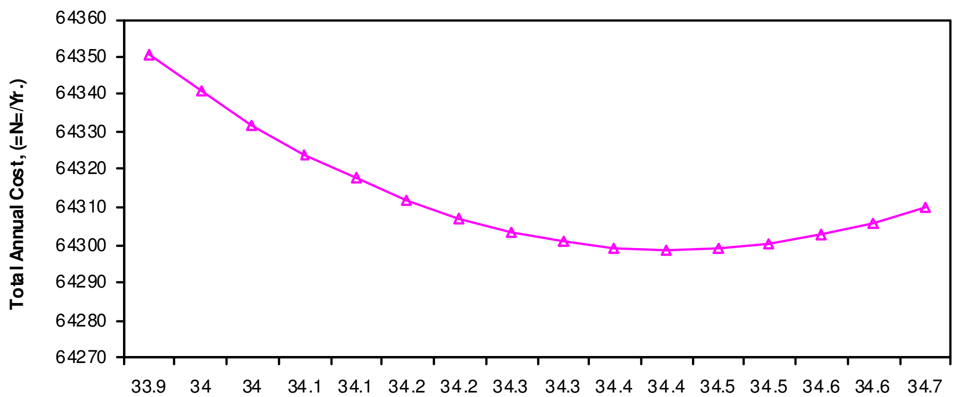

***Figure 1**. Variation of total annual cost with pipe sizes for a turbulent flow case study (program output)*

The results obtained from the computer program for different values of compressibility are shown in Table 3.

Table 3. Program Results for Different Fluid Compressibility.

S /N	Compressibility X 10^3(m^3kg^{-1})	Optimum Pipe Diameter(mm)
1	1.136363	34.400
2	1.111111	34.150
3	1.098901	34.050
4	1.086956	33.950
5	1.075262	33.800
6	1.063829	33.700
7	1.052632	33.600
8	1.041666	33.455
9	1.030928	33.350
10	1.020408	33.250
11	1.010101	33.150
12	1.000000	33.050

Discussion

From Table 2 and Fig.1, the total annual cost decreases to a minimum value of N64, 298.60 with a corresponding pipe diameter of 34.40mm and then steadily increases. The recommended pipe diameter therefore, will be the one corresponding to the lowest total annual costs. The results obtained are in good agreement with an otherwise method of differential calculus. The approach involves differentiation of equation (9) with respect to diameter, d, equating the resulting expression to zero, and solving the expression for the optimum diameter.

A plot of the optimum pipe diameter against compressibility, representative of different fluids, is shown in Fig.2. It can be inferred from Fig.2 that the economic pipe diameter increases linearly with fluid compressibility. Conversely, this implies that less dense fluids have larger optimum pipe diameter.

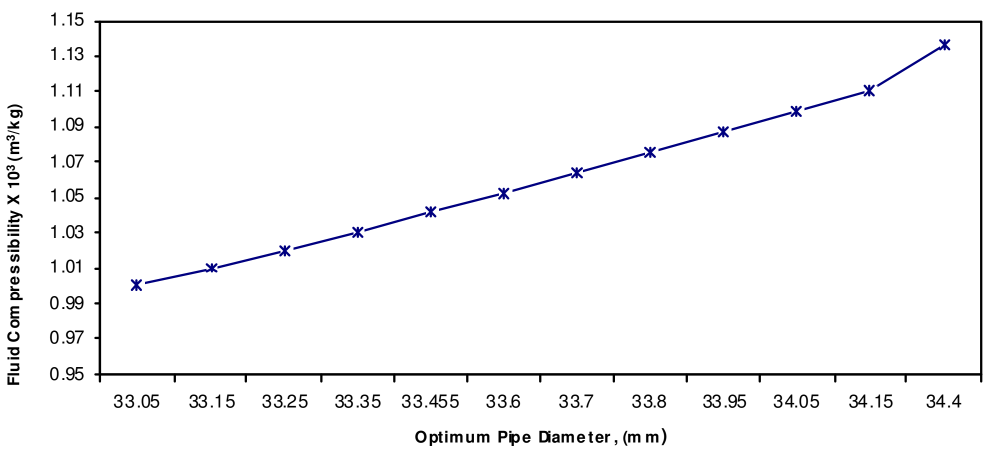

Figure 2. *Variation of optimum pipe diameter with fluid compressibility for turbulent flow*

Conclusions

A computer aided technique for determination of optimum pipe diameter for non – viscous flow was developed using the least cost approach. Results obtained from the validation of the developed software revealed that optimum pipe diameter for turbulent flow increases linearly with fluid compressibility.

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Appendix

Computer Program

```
//PROGRAM:OPTIMUM PIPE DIAMETER FOR TURBULENT FLOW^^^^^^^^^^^^^/
//COMPILER: MICROSOFT VISUAL C++ 6.0^^^^^^^^^^^^^^^^/
#include<iostream.h>
#include<fstream.h>
```

```
#include<math.h>
float c,e,f,n,E,CMP;
char filename[25];
double Cpiping,Cp,Cf,x,y,Ce,g,H,Q,N,Cpipe,F,a,b,Re,Ct,d,D;
int main (){
cout <<" supply filename"<< endl;
cin >>filename;
ofstream fout(filename);
float deltaD,N,Ce,a,b,F;
cout<<" enter fluid flow rate, (kg/s)\n";
cin>>Q;
cout<<"enter fluid COMPRESSIBILITY, (cubic metre/kg)\n";
cin>>CMP;
cout<<"Supply fluid viscosity, (Ns/sq.metre)\n";
cin>>N;
cout<<"enter cost of electrical energy, (#/kWhr)\n";
cin>>Ce;
cout<<"supply Hours of operation per year, (hr./yr.)\n";
cin >>H;
cout<<"enter  the initial value of pipe diameter (mm)"<<endl;
cin>>D;
cout<<"Supply efficiency of compressor and pump, (decimal)\n";
cin>>E;
cout<<"enter the small increment in Pipe diameter, (mm)"<<endl;
cin>> deltaD;
cout<<"enter the value of maintenance charge, (per cent)\n";
cin>>b;
cout<<"enter total cost of valves,fittings and erection,"
<< "(ratio of cost of new pipe)\n";
cin>>F;
cout<<"Supply purchase cost for new one milimetre diametre per"
<<"milimetre of pipe length, (#/mm)\n";
```

```
cin>>Cpipe;
cout<<"Supply the value of capital charge, (per cent)\n";
cin>>a;
cout<<"Inside Diameter of pipe Total annual cost"<<endl;
fout<<"Inside Diameter of pipe Total annual cost"<<endl;
for(int i=0; i<55; i++){
if(D>=25.0)
n=1.5;
else
n=1.0;
d=pow(D,n);
cout<<"d"<<d;
e=(a+b);
cout<<"e"<<e;
f=(1+F);
cout<<"f"<<f;
c=e*f;
cout<<"c"<<c;
g=pow(CMP,(2))/E;
cout<<"g"<<g;
y=4.13e10*Ce*g;
cout<<"y"<<y;
Cpiping=d*c*Cpipe;
cout<<"Cpiping"<<Cpiping;
x=pow(N,0.16)*y;
cout<<"x"<<x;
Cf=H*pow(Q,2.84)*x;
cout<<"Cf"<<Cf;
Cf=Cf*pow(D,(-4.84));
Re = 4.0*Q/CMP;
cout<<"Re"<<Re;
Re = Re/(3.141592654*N*D);
```

```
cout<<"Re"<<Re;
if(Re>=2000.0) {
Cp = Cf;
cout<<"Cp"<<Cp;
Ct = Cp+Cpiping; }
else {
cout<<"Flow is not turbulent"<<endl;
}
cin>>Ct;
D+=deltaD;
cout<<D<<"\t"<<"\t"<<"\t"<<"\t"<<"\t"<<"\t"<<"\t"<<"\t"<<Ct<<endl;
fout<<D<<"\t"<<"\t"<<"\t"<<"\t"<<"\t"<<"\t"<<"\t"<<"\t"<<Ct<<endl;
}
return 0;
}
```