Optimal sensors placement for monitoring a steam condenser of the distillation column using bond graph approach

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Abstract
This paper deals with monitoring of a process engineering system. The steam condenser was monitored by bond graph tool. The model was constituted by nine capacitive and resistive elements which needed minimum of sensors. This method was based on Analytical Redundancy Relations which were generated from a condenser model and represented residuals. After substitution, we obtained the placement of six sensors which guaranteed the monitoring of nine components. A fault is created by the abrupt annulment of the fluid flow value provided by the source. The block diagram is elaborated on SYMBOLS software and we supervised the residuals evolution.

Keywords
Optimization; Sensors placement; Monitoring; Condenser; Distillation Column; Bond graph

Introduction
The monitoring system with rigid structure knows progress during the last years. The monitoring of the thermal system is difficult because of the phenomenon complexity [1]. These industrial processes have a strongly nonlinear behaviour due mainly to the mutual
interaction of several phenomena with various nature and the association of the technological components. The system modelled and monitored, is a condenser of the distillation column which is produced thermal, chemical and hydraulic phenomena. In this case, the bond graph tool reduces all difficulties because of its causal and structural properties. After the validation of the model, the placement of the sensors, is made and followed by the research of the optimal case [2], and [3]. The advantages of applying bond graph model to the condenser of distillation column are the ability to build a graphical circuit, the use of detailed equations, the encapsulation of pieces of the model into sub-models and the representation of the system power flows in details. The main drawback of using bond graphs is that they cannot describe distillation equations. In this work, both bond graph modelling and monitoring by sensors placement are proposed for a condenser of a distillation column.

**Description of the system**

The condenser schematic is shown in Figure 1. The steam enters at the left side, condensed on the pipes filled with cooling water.

It falls to the floors and leaves from there to the receiver through control valves [1]. The water outflow (condensate) is controlled by three valves in order to keep its level constant. The condenser is a module of four input-outputs. It is composed by a coolant circuit made up of a unit of pipes which lead the liquid. The liquid phase of cold fluid accumulates heat while passing in steam (vapor) phase. When the steam flow attempts the condenser, a contact is obtained between the steam and the pipes of the fluid circuit in which circulate both the cold fluid and the steam condensed in fine droplets which are formed at the condenser bottom [4].

![Figure 1. The condenser operating in the distillation column](image)
Steam circulating in the column is from a portion of the feed when it is vaporized. In
the boiler, the liquid mixture of the beginning is heated for the vaporizer. The steam which
arrives in the line of column heading is condensed completely or partially by a condenser [5],
[6]. When the steam arrives in contact with the fluid pipes in which circulates a cold fluid, it
was condensed in liquid which runs out downwards by simple gravity and collected at the
bottom of the condenser. The liquid obtained is generally returned in the column under the
name of backward flow by tubular of feed. The other part is collected like a distillate and the
non-vaporized part comes to contribute to the liquid flow. The operation is repeated, by
ensuring the condensation of the steam of column heading [2].

Sensors placement algorithm

This procedure consists of an optimal sensors placement for the components
monitoring, i.e. to detect and to isolate the components failures. From a physical process we
elaborate a bond graph model. In bond graph model, sensors are placed only at the junction 0
and 1, we consider a virtual placement of the sensor at the position j, which is represented by
the variables x and y. N0, and N1 are respectively the junction numbers 0Ci, 1Rj. i, j, n and m are
respectively the junction orders, the number of the bonds’ fixed on the junction 0Ci, 1Rj. The
structural equations of the junction s0Ci, which describe the effort equality and the pressure
conservation produced in C element, are given by (1).

\[
\begin{align*}
\sum_{k=1}^{n} a_k f_k &= 0 \\
e_k &= e_{Ci}
\end{align*}
\] (1)

where \(f_k\) and \(e_k\) are respectively the flow and the effort of the junction number \(k\); \(a_k=+1\) if the
bond graph semi flow is toward junction; \(a_k=-1\) otherwise. For the nonlinear functions \(\Phi_{Ci}\) and
s the Laplace coefficient used here as the derived operator, the flow and the effort variables are
determined by (2) as follows:

\[
\begin{align*}
f_{Ci} &= \phi_{Ci} \left[ s \left( 1-x_i \right) e_{Ci} + x_i D e_i \right] \\
e_{Ci} &= \frac{1}{s} \left( 1-x_i \right) \phi_{Ci}^{-1} (f_{Ci}) + x_i D e_i
\end{align*}
\] (2)

where \(i=1, N_0\); \(D e_i\) effort detectors, \(x_i\) binary variables \(x_i=1\) if we place detector; \(x_i=0\)
otherwise. The structural equations of the junctions 1Rj are given by equation (3).
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\[
\begin{aligned}
\sum_{i=1}^{n} a_i e_i &= 0 \\
f_i &= f_{\bar{r}_i}
\end{aligned}
\]  

(3)

For the nonlinear functions \( \Phi_{Rj} \), the flow and effort variables are determined by (4):

\[
\begin{aligned}
e_{\bar{r}_j} &= \phi_{Rj} \left( \left(1 - y_j\right)f_{\bar{r}_j} + y_j Df_j \right) \\
f_{\bar{r}_j} &= \left(1 - y_j\right)\phi_{Rj}^{-1} e_{\bar{r}_j} + y_j Df_j
\end{aligned}
\]  

(4)

where \( j = 1, N_j; Df_j \) flow detectors, \( y_j \) binary variables \( y_j = 1 \) if we place detector; \( y_j = 0 \) otherwise. The combinations of the variables \( x_i \) and \( y_j \) make it possible to generate analytical redundancy relations which give the structures of the residuals. The bond graph model of the condenser in the distillation column representing hydraulic and thermal phenomena with steam and liquid phases and using sensors placement (effort and flow detectors) is represented by Figure 2. This model is composed by 5 junctions type 0 attached to 5 components \( CV, CL, CT_1, CT_3 \) and \( CT_5 \) noted \( 0_1, 0_2, 0_3, 0_4, 0_5 \) and 4 junctions type 1 attached to 4 components \( RT_1, RT_3, RT_5 \) and \( R \) noted \( 1_1, 1_2, 1_3, 1_4 [2] \). These components are: \( CV \) the pipe capacity traveled by fluid in vapor phase, \( CL \) the pipe capacity of fluid in liquid phase; \( CT_1 \) the tank capacity; \( CT_3 \) the tank3 capacity; \( CT_5 \) the tank capacity of a distillate; \( RT_1, RT_3, RT_5 \) and \( R \) are respectively, the valves of tank1 and tank3, the distillate and the reflux drum.

Table 1. Nomenclature

<table>
<thead>
<tr>
<th>Variables</th>
<th>Bond graph descriptions</th>
<th>Physical descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>Effort</td>
<td>Pressure ( P ), Temperature ( T )</td>
</tr>
<tr>
<td>F</td>
<td>Flow</td>
<td>Masse flow ( dm/dt ), Enthalpy flow ( dH/dt )</td>
</tr>
<tr>
<td>De</td>
<td>Effort detector</td>
<td>Measured effort</td>
</tr>
<tr>
<td>Df</td>
<td>Flow detector</td>
<td>Measured flow</td>
</tr>
<tr>
<td>Se, MSE</td>
<td>Effort source</td>
<td>Thermal or Hydraulic effort</td>
</tr>
<tr>
<td>Sf, MSf</td>
<td>Flow source</td>
<td>Thermal or Hydraulic flow</td>
</tr>
<tr>
<td>R</td>
<td>Resistive component</td>
<td>Valve</td>
</tr>
<tr>
<td>C</td>
<td>Capacitive component</td>
<td>Tank</td>
</tr>
<tr>
<td>ARR</td>
<td>Analytical Redundancy Relation</td>
<td>Residual (Failure indicators)</td>
</tr>
<tr>
<td>S</td>
<td>Derivative operator</td>
<td>( d/dt )</td>
</tr>
<tr>
<td>1/s</td>
<td>Integral operator</td>
<td>( \int dt )</td>
</tr>
<tr>
<td>( \Phi )</td>
<td>Nonlinear components evolution</td>
<td>( \Phi_R; \Phi_{CV}; \Phi_{CL}; \Phi_{CT1}; \Phi_{CT3}; \Phi_{CTS}; \Phi_{RT1}; \Phi_{RT3}; \Phi_{RTS} )</td>
</tr>
<tr>
<td>( CV )</td>
<td>Vapour (steam) capacity</td>
<td>Pipe capacity travelled by fluid in vapour phase</td>
</tr>
<tr>
<td>( CL )</td>
<td>Liquid capacity</td>
<td>Pipe capacity travelled by fluid in liquid phase</td>
</tr>
<tr>
<td>( CT_1 )</td>
<td>Tank capacity</td>
<td>Level of fluid into tank1</td>
</tr>
<tr>
<td>( CT_3 )</td>
<td>Tank3 capacity</td>
<td>Level of fluid into tank3</td>
</tr>
<tr>
<td>( CT_5 )</td>
<td>Capacity of distillate tank</td>
<td>Distillate into reflux drum</td>
</tr>
<tr>
<td>( RT_1 )</td>
<td>Tank1 resistor</td>
<td>Valve of fluid into tank1</td>
</tr>
<tr>
<td>( RT_3 )</td>
<td>Tank3 resistor</td>
<td>Valve of fluid into tank3</td>
</tr>
<tr>
<td>( RT_5 )</td>
<td>Resistor of distillate tank</td>
<td>Distillate valve</td>
</tr>
</tbody>
</table>
The bond graph model is described as follows:

\[ e_1 = e_2 = e_3 \]
\[ f_3 - f_1 - f_2 = 0; f_1 = Msf_1; f_2 = Msf_2 \]
\[ f_{CV} = f_3 = \phi_{CV} \left[ s \left( 1 - x_3 \right) e_3 + x_3 De_1 \right] \]
\[ e_{CV} = e_3 = 1/s \left( 1 - x_3 \right) \phi_{CV}^{-1} \left( f_3 \right) + x_3 De_1 \]

\[ f_{10} = f_{12} = f_{15} \]
\[ e_{12} - e_{10} - e_{15} = 0 \]
\[ e_{12} = \phi_{RT1} \left[ \left( 1 - y \right) f_{12} + y_1 Df_1 \right] \]
\[ f_{12} = f_{RT1} = \left( 1 - y \right) \phi_{RT1}^{-1} e_{12} + y_1 Df_1 \]
where $Msf_1$ and $Msf_2$ represent the controlled (modulated by signals) flow source of bond 1 and bond 2. From the previous equations we can deduce the system (7) which permits us to generate residuals:

\[
\phi_{cr} \left[ s \left\{ (1 - x_1) e_3 + x_1 De_1 \right\} \right] - Msf_1 - Msf_2 = 0
\]
\[
\phi_{cT_1} \left[ s \left\{ (1 - x_2) f_2 + y_2 Df_2 \right\} \right] - \left\{ (1 - x_3) M_{T_1}^1 f_4 + x_2 De_2 \right\} - \left\{ (1 - x_3) M_{T_1}^1 f_4 \right\} - x_3 De_3 = 0
\]
\[
\phi_{cT_2} \left[ s \left\{ (1 - x_4) e_4 + x_4 De_4 \right\} \right] - \left\{ (1 - x_4) y_4 Df_4 \right\} - \left\{ (1 - x_4) y_4 Df_4 \right\} + y_2 Df_2 = 0
\]
\[
\phi_{cT_4} \left[ s \left\{ (1 - x_5) e_5 + x_5 De_5 \right\} \right] - \left\{ (1 - x_5) y_5 Df_5 \right\} - \left\{ (1 - x_5) y_5 Df_5 \right\} + Df_i = 0
\]
\[
\phi_{cT_3} \left[ s \left\{ (1 - x_6) e_6 + x_6 De_6 \right\} \right] - \left\{ (1 - x_6) y_6 Df_6 \right\} - \left\{ (1 - x_6) y_6 Df_6 \right\} + Df_i = 0
\]
\[
\phi_{cT_3} \left[ s \left\{ (1 - x_7) e_7 + x_7 De_7 \right\} \right] - \left\{ (1 - x_7) y_7 Df_7 \right\} - \left\{ (1 - x_7) y_7 Df_7 \right\} + Df_i = 0
\]
\[
\phi_{cl} \left[ s \left\{ (1 - x_8) e_8 + x_8 De_8 \right\} \right] - \left\{ (1 - x_8) y_8 Df_8 \right\} - \left\{ (1 - x_8) y_8 Df_8 \right\} + Df_i = 0
\]

(7)

**Optimization technique**

The equations above allow according to binary variables $x_i$ and $y_j$ determining the final system structure to supervise. The optimal case is with how many sensors to supervise this model. There exist $2^n = 2^9 = 512$ combinations to obtain the optimal case.

From the combination with 9 sensors placed in the 9 positions, the ARRs are calculated until to obtain the optimal combination. The detection is defined by columns of the signature table different to zero. The isolation is guaranteed by the columns different each other.

In the second step, the substitution is used to reduce the number of ARRs and to verify the monitoring from the signature table. In our case, 6 ARRs satisfied that all the components are monitored. The generated ARRs are given by the following relations (the sensors are placed only if $x_i$ and $y_j = 1$).

**Substitution with 9 sensors**

All the sensors are placed, so each component is monitored by one sensor and the system (7) becomes as follows for $\{x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4, x_5, x_6, x_7\}^T = [1101111110]^T$

\[
ARR_1 = \phi_{cr} \left[ s De_1 \right] - Msf_1 - Msf_2
\]

(8)
\[
\begin{align*}
\text{ARR}_1 &= \phi_{\alpha T_1}[Df_1] - \phi_{CV}^{-1}[Msf_3 + Msf_6 - Df_2 - Df_3 - Df_7] \\
\text{ARR}_2 &= \phi_{\alpha T_2}[Df_7] - \phi_{CV}^{-1}[Msf_3 + Msf_6 - Df_2 - Df_3 - Df_7] - De_4 - De_6 \\
\text{ARR}_3 &= \phi_{\alpha T_3}[Df_3] - \phi_{CV}^{-1}[Msf_3 + Msf_6 - Df_2 - Df_3 - Df_7] - De_3 \\
\text{ARR}_4 &= \phi_{\alpha T_4}[Df_7] + Df_2 + Df_3 - De_4 \\
\text{ARR}_5 &= \phi_{\alpha T_5}[Df_4] - De_5 + De_1 \\
\text{ARR}_6 &= \phi_{\alpha T_6}[Df_4] - De_5 + Df_5 \\
\text{ARR}_7 &= \phi_{\alpha T_7}[Df_4] + Df_5 + De_5 \\
\text{ARR}_8 &= \phi_{\alpha T_8}[Df_4] + Df_5 - Msf_{30} - Msf_{37} \\
\text{ARR}_9 &= \phi_{\alpha T_9}[Df_4] - Df_2 - Df_3 - Msf_{30} - Msf_{37} \\
\end{align*}
\]

**Table 2. Faults signature table for 9 detectors**

<table>
<thead>
<tr>
<th></th>
<th>C_V</th>
<th>C_L</th>
<th>C_T1</th>
<th>R_T1</th>
<th>C_T2</th>
<th>R_T2</th>
<th>C_T3</th>
<th>R_T3</th>
<th>C_T4</th>
<th>R_T4</th>
<th>R_T5</th>
<th>R_T6</th>
<th>R_T7</th>
<th>R_T8</th>
<th>R_T9</th>
<th>R_T10</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARR_1</td>
<td>1</td>
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<td>ARR_2</td>
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<td>ARR_4</td>
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<td>ARR_5</td>
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<td>ARR_6</td>
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<td>ARR_7</td>
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<td>ARR_8</td>
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</tbody>
</table>

In this case faults on all the components are detectable and localizable.

**Substitution with 8 sensors**

One of the possible combinations is chosen, so 8 ARRs are generated for monitoring 9 components. The combination used in the system (7) is \([011011110]_{10}\).

\[
\begin{align*}
\text{ARR}_1 &= \phi_{\alpha T_1}[Df_1] - De_2 - De_3 \\
\text{ARR}_2 &= \phi_{CV}[sDf_3] + Df_1 + Df_3 - Msf_3 - Msf_6 \\
\text{ARR}_3 &= \phi_{CT_3}[sDf_3] + Df_2 + Df_3 \\
\text{ARR}_4 &= \phi_{RT_3}[Df_3] - De_2 - De_3 \\
\text{ARR}_5 &= \phi_{RT_4}[sDf_3] + Df_3 + Df_4 + De_5 \\
\text{ARR}_6 &= \phi_{RT_5}[Df_3] + Df_3 + De_4 + De_5 \\
\text{ARR}_7 &= \phi_{RT_6}[sDf_3] + Df_3 + Df_4 + De_5 \\
\text{ARR}_8 &= \phi_{RT_7}[sDf_3] + Df_3 + De_4 + De_5 \\
\end{align*}
\]
Table 3. Faults signature table for 8 detectors

<table>
<thead>
<tr>
<th></th>
<th>C_V</th>
<th>C_L</th>
<th>C_T1</th>
<th>R_T1</th>
<th>C_T3</th>
<th>R_T3</th>
<th>C_T5</th>
<th>R_T5</th>
<th>R</th>
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<tbody>
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<td>ARR_1</td>
<td>1</td>
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<td>ARR_2</td>
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<td>ARR_3</td>
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<td>ARR_4</td>
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<td>ARR_8</td>
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<td></td>
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</tr>
</tbody>
</table>

In this case faults on all the components are detectable and isolable.

Substitution with 7 sensors

Not localizable case $[01000111111]^T$

\[
\begin{align*}
\text{ARR}_1 &= \phi_{CT1}[Df_1] - \frac{1}{z} \phi_{CT3}^{-1} [Df_3 - Df_7] - \frac{1}{z} \phi_{CT1}^{-1} [Df_2] \\
\text{ARR}_2 &= \phi_{CT3}[sDe_4] + \phi_{RT3}^{-1} [De_4 + De_5] + Df_3 - Df_4 \\
\text{ARR}_3 &= \phi_{C_2}[Df_4] - Df_5 + Df_4 + Df_5 \\
\text{ARR}_4 &= \phi_{CT3}[sDe_3] + Df_5 + Df_4 + Df_5 \\
\text{ARR}_5 &= \phi_{CT}[sDe_6] + Df_2 + Df_3 - Msf_{35} - Msf_{37} \\
\text{ARR}_6 &= \phi_{C_2}[sDe_5] - Msf_{36} - Msf_{41} \\
\text{ARR}_7 &= \phi_{RT}[Df_3] - \frac{1}{z} \phi_{CV}^{-1} [Df_3 - Df_7] - Df_3
\end{align*}
\]

(10)

Table 4. Faults signature table for 7 detectors

<table>
<thead>
<tr>
<th></th>
<th>C_V</th>
<th>C_L</th>
<th>C_T1</th>
<th>R_T1</th>
<th>C_T3</th>
<th>R_T3</th>
<th>C_T5</th>
<th>R_T5</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARR_1</td>
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<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ARR_2</td>
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<td></td>
<td>1</td>
<td>1</td>
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</tr>
<tr>
<td>ARR_3</td>
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</tr>
<tr>
<td>ARR_4</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
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<td></td>
</tr>
<tr>
<td>ARR_5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ARR_6</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>ARR_7</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

The fault on (C_{T1} and R_{T1}) and the fault on (C_{T3} and R_{T3}) are not localizable (the same column vectors).

Detectable and isolable case $[00101111110]^T$

\[
\begin{align*}
\text{ARR}_1 &= \phi_{CT1}[sDe_2] - \phi_{RT1}^{-1} [De_2 + De_3] - Sf_5 - Sf_6 - Df_7^* \\
\text{ARR}_2 &= \phi_{CT1}[sDe_1] + Df_1^* + Df_2^* - Df_1^*
\end{align*}
\]

(11)
\[ ARR_1 = \phi_{RTS} [Df_3] - De_2 - De_5 \]
\[ ARR_4 = \phi_{CT3} [sDe_4] + \phi_{RTS}^{-1} [De_2 + De_3] + Df_3 - Df_4 \]
\[ ARR_3 = \phi_{R} [Df_4] - De_5 + Df_1^* \]
\[ ARR_6 = \phi_{CT3} [sDe_1] + Df_3 + Df_4 + Df_5^* \]
\[ ARR_5 = \phi_{CL} [sDe_5] + \phi_{RT3}^{-1} [De_4 + De_6] - Sf_{3c} - Sf_{3d} + Df_6^* \]

**Table 5.** Faults signature table for 7 detectors

<table>
<thead>
<tr>
<th></th>
<th>CV</th>
<th>CL</th>
<th>CT1</th>
<th>RT1</th>
<th>CT3</th>
<th>RT3</th>
<th>CTS</th>
<th>RTS</th>
<th>R</th>
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<tr>
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<td></td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
The table (6) of the residuals structure permit to notice that the residuals structures are different and the faults signatures are different and not equal to zero, thus the components $C_V$, $C_L$, $C_T1$, $R_T1$, $C_T3$, $R_T3$, $C_TS$, $R_TS$ and $R$ are monitored.

**Sensitivity of the detectors**

Under normal operating, the residuals must be constantly equal to zero but it is not always the case because of the modelling errors and the unavailability of the technical specifications of the real system.

From SYMBOLS software (System Modelling Bond graph Language Simulation), we have implanted the bond graph model [5] associated to the block diagram of the ARR that contain affected components.

We create fault on $R_T1$ component monitored by the detector $Df_1$ (disturbance from the association of Dirac pulse and Unit echelon). Their values are not considered only their appearances in the relation are taking into account with evaluation term (1 for the presence and 0 for the absence).

In the first time, the fault is created between the instant $t_1=2s$ and $t_2=2.01s$ by the annulment of fluid flow provided by the source [9], [10].

In the second time, the block diagram of the $ARR_1$, $ARR_2$ and $ARR_3$ expressions that contain $Df_1$ detector (see Figure. 3) is elaborated. The three outputs are represented by three scopes.

The failure is injected on $I_{RT1}$ junction monitored by the detector $Df_1$ (Figure. 4) and not sensitive to $ARR_4$, $ARR_5$ and $ARR_6$ (Figure. 5). It represents sealing in the valve and leakage in the tank.
Figure 3. Block diagram representing the ARRs function of Df$_i$

Figure 4. Sensitivity of Df$_i$ detector with failed operating of ARR$_1$, ARR$_2$ and ARR$_3$
Sensors placement for monitoring a steam condenser of the distillation column using bond graph approach

Samia LATRECHE, Mohammed MOSTEFAI, Mabrouk KHEMLICHE

Figure 5. Sensitivity of Df1 detector with normal operating of ARR4, ARR5 and ARR6

The residuals ARR1, ARR2 and ARR3 are sensitive to the failures, it is due to the fact that the Df1 detector appears strongly in these residuals, on the other hand residuals ARR4, ARR5 and ARR6 are null because the detector does not appear in the relations of these residuals it is the normal operating.

Conclusions

The bond graph tool used for modelling and monitoring is very efficient because of the complexity of the phenomena that are produced in the steam condenser as hydraulic and thermal phenomena and the separation between steam and liquid phases.

The Analytical Redundancy Relations are deduced from graphical model of the steam condenser of the distillation column.

The structural junction equations and constitutive element laws for generating these relations are used as the failures indicators.

The algorithm used for researching the optimal case is very efficient.
The simulator of Symbols Software allows the validation and simulation of the model. The technique applied to steam condenser for components monitoring can be extended to detect and locate failures actuators and sensors.

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References

