

Application of PQR theory for control of a 3-phase 4-wire 4-legs shunt active power filter in the $\alpha\beta$ -axes using 3d-svm technique

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Abstract

This article discusses and compares two control strategies applied to a 3-phase 4-wire 4-leg shunt active power filter. These two control strategies, including the cross-vector theory called CV theory and the direct method called PQR theory, are based on the instantaneous control of active and reactive power. On one hand, it is shown that, in some cases, cross-vector theory requires elimination of the zero sequence currents in a 3-phase 4-wire 4-leg shunt active power filter, which needs a power storage element, and on the other hand pretreatment system voltage is necessary to obtain compensated sinusoidal current and a degree of freedom. By relying on the cross-vector theory, the PQR theory is used to extract and remove harmonic currents components. In this control technique, there are two internal current control loops and an external voltage control loop, these control loops have been realized by PI controllers when applied 3D-SVM of switching technique. We choose as criteria for comparison the transient and the Total Harmonic Distortion in the line current. A series of simulations in MATLAB/ Simulink environment have been presented and discussed to show the performance of the two control strategies.

Keywords

3-phase 4-wire 4-leg SAPF; CV theory; PQR theory; Harmonic currents

Introduction

The development of systems based on power electronics leads to an increasing number of network-connected non-linear loads. This has been applied in both in industry (motor drive, rectifier) and in the domestic sector (computer, TV) [1]. These non-linear loads absorb non-sinusoidal currents, which lead both to the appearance of a disturbing phenomenon which is becoming more and more extensive, causing genes and even malfunctions in electrical equipment, and to the worries of the distributors 'electrical energy [2].

This is nothing other than the harmonic distortion. The result is the presence and spread throughout the network harmonic currents, resulting in most cases odd rows. This is why the improvement of the electrical energy is important and it is interesting to find solutions in order to improve further but cleaner and more durable.

In view of these findings, it therefore seems important to limit the propagation of these harmonic currents to ensure satisfactory quality of energy. It is with this aim that power electronic devices should be used to help clean up the network. This could be achieved by using the inverter as an active power filter by adding references of harmonic line currents to the command which already controls the powers [3].

This solution consists of implementing a filter called active, the principle was proposed in the early eighties [4,5]. This device is a power converter which injects in the electricity grid harmonic currents equal to those absorbed by the load, but in opposite phase with them. Thus, the filter is constantly adapting to the harmonic currents on the network and allows for compensation of all harmonics. These active power filters come in different structures and there is a wide variety of commands depending on whether one wants to correct voltages (series filter) or current (parallel filter) [3]. This is currently the most effective solution.

Several studies have shown that the conventional three-phase active power filter cannot compensate the disturbances due to single-phase non-linear loads connected to a four-wire network. [6]. Thus, the main objectives of this work was to study in detail the technical control of shunt active power filter with four arms to eliminate the harmonic of currents

flowing in the network and the zero-sequence current [7], by relying on the cross-vector theory, which was used for extracted and removing harmonic currents. The PQR method was introduced to eliminate the zero-sequence current away. In the PQR theory the independence between the three variables p_{lp} , p_{lq} and p_{lr} offers a degree of freedom equal to 3, this means that it is possible to set each power variable without affecting other variables [8].

Eventually, to ensure a good performance, the conventional PI controllers was used when applied 3D-SVM of switching technique to control the 4-legs inverter. The simulation results of the two studied methods will be presented in order to prove the efficiency of them.

Materials and method

Four arms shunt active power filter

The underlying principle of four arms shunt active power filter is that the current load is sampled and calculated fast. As for that the harmonic current is caught (Figure 1). Through the compensation current generating circuits, the load harmonic current is rejected in the grid connection point of power system and load, so that the supply current is sinusoidal. The Four arms shunt active power filter is an arbitrary current generator in fact, which can generate the load harmonic currents so that the input current of required power is sinusoidal [9].

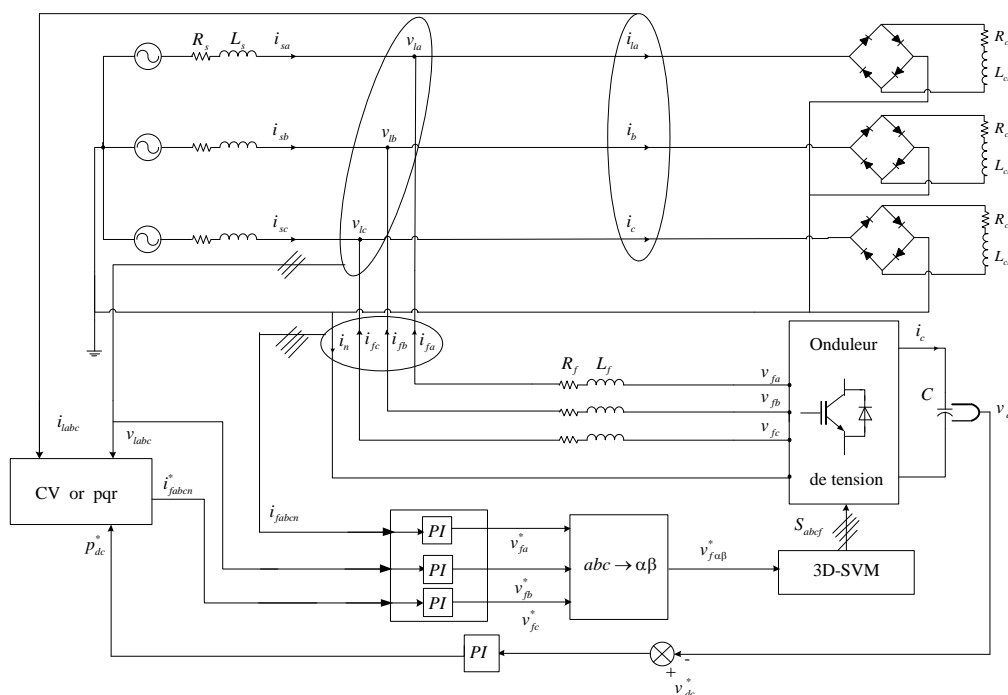


Figure 1. The main circuit of the 3-phase 4-wire 4-legs shunt active power filter

Algorithms of extracted and removing harmonic currents

The cross-vector theory

Cross-vector theory defines instantaneous active and reactive power respectively by the scalar and vector product of the voltage and the current space vectors in the time domain in a three-phase four-wire system [6, 10]. Here, the instantaneous active and reactive powers have the same physical meaning of the instantaneous real and imaginary powers in pq theory. The authors prefer to express it as instantaneous real and imaginary power to distinguish it from the traditional definition of active and reactive power in the frequency domain.

$$\left\{ \begin{array}{l} \frac{di_{f\alpha}}{dt} = -\frac{R_f}{L_f} i_{f\alpha} + \frac{1}{L_f} v_{f\alpha} - \frac{1}{L_f} v_{l\alpha} \\ \frac{di_{f\beta}}{dt} = -\frac{R_f}{L_f} i_{f\beta} + \frac{1}{L_f} v_{f\beta} - \frac{1}{L_f} v_{l\beta} \\ \frac{di_{fo}}{dt} = -\frac{R_f}{L_f} i_{fo} + \frac{1}{L_f} v_{fo} - \frac{1}{L_f} v_{lo} \\ \frac{dv_{dc}}{dt} = \frac{P_{dc}^*}{C} \end{array} \right. \quad (1)$$

Figure 2 present schematic principal of the cross-vector theory [11].

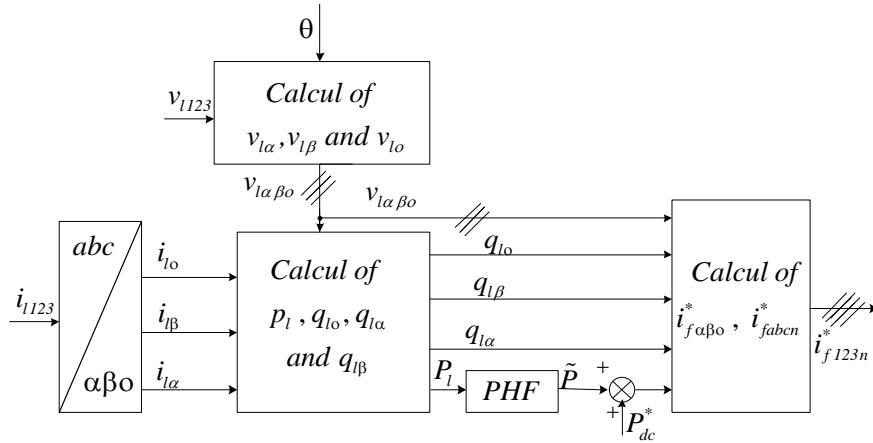


Figure 2. Schematic diagram of the cross-vector theory (Adapted from [11])

The instantaneous power is calculated as follows:

$$\begin{bmatrix} P_l \\ q_l \\ P_{l\alpha} \\ q_{l\beta} \end{bmatrix} = \begin{bmatrix} v_{lo} & v_{l\alpha} & v_{l\beta} \\ 0 & -v_{l\beta} & v_{l\alpha} \\ v_{l\beta} & 0 & -v_{lo} \\ -v_{l\alpha} & v_{lo} & 0 \end{bmatrix} \begin{bmatrix} i_{l\alpha} \\ i_{l\beta} \\ i_{lo} \end{bmatrix} \quad (2)$$

From equation (2), we can deduce the corresponding current components:

$$\begin{bmatrix} i_{l\alpha} \\ i_{l\beta} \\ i_{l0} \end{bmatrix} = \begin{bmatrix} v_{l0} & v_{l\alpha} & v_{l\beta} \\ 0 & -v_{l\beta} & v_{l\alpha} \\ v_{l\beta} & 0 & -v_{l0} \\ -v_{l\alpha} & v_{l0} & 0 \end{bmatrix}^{-1} \begin{bmatrix} p_l \\ q_{l0} \\ p_{l\alpha} \\ q_{l\beta} \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} i_{l0} \\ i_{l\alpha} \\ i_{l\beta} \end{bmatrix} = A' \begin{bmatrix} v_{l0} & 0 & v_{l\beta} & v_{l\alpha} \\ v_{l\alpha} & -v_{l\beta} & 0 & v_{l0} \\ v_{l\beta} & v_{l\alpha} & -v_{l0} & 0 \end{bmatrix} \begin{bmatrix} p_l \\ q_{l0} \\ q_{l\alpha} \\ q_{l\beta} \end{bmatrix} \quad (4)$$

with: $A' = \frac{1}{v_{l\alpha}^2 + v_{l\beta}^2 + v_{l0}^2}$

Table (1) summarizes the modes of possible compensations:

Table 1. Modes of compensation of the cross-vector theory

Type of compensation	Parameter of control
Compensation of the harmonic of currents	$p_f^* = \tilde{p}_l$ et $q_{fx}^* = \tilde{q}_x$, with $x = \alpha, \beta, 0$
Compensation of reactive power	$p_f^* = 0$ et $q_{f0}^* = \bar{q}_{l0}$
Compensation of the harmonic of currents and reactive power	$p_f^* = \tilde{p}_l$ et $q_{fx}^* = q_{lx}$, with $x = \alpha, \beta, 0$

We selected to compensate for at the same time the harmonics of current and reactive energy, the equation (4) will thus become:

$$\begin{bmatrix} i_{f0}^* \\ i_{f\alpha}^* \\ i_{f\beta}^* \end{bmatrix} = A' \begin{bmatrix} v_{l0} & 0 & v_{l\beta} & v_{l\alpha} \\ v_{l\alpha} & -v_{l\beta} & 0 & v_{l0} \\ v_{l\beta} & v_{l\alpha} & -v_{l0} & 0 \end{bmatrix} \begin{bmatrix} p_f^* \\ q_{f0}^* \\ q_{f\alpha}^* \\ q_{f\beta}^* \end{bmatrix} \quad (5)$$

with: $p_f^* = \tilde{p}_l$, $q_{f0}^* = q_{l0}$, $q_{f\alpha}^* = q_{l\alpha}$, $q_{f\beta}^* = q_{l\beta}$.

The inverse Concordia transform calculates the reference currents in the *abc*-axes as follows:

$$\begin{bmatrix} i_{f1}^* \\ i_{f2}^* \\ i_{f3}^* \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_{f\alpha}^* \\ i_{f\beta}^* \\ i_{fo}^* \end{bmatrix} \quad (6)$$

The PQR theory

The PQR theory is effected by double transformation process [7]. A first transformation of voltages and line currents from the abc-axes to $\alpha\beta o$ -axes, then a second from the $\alpha\beta o$ -axes to the pqr-axes [8,9].

$$\begin{bmatrix} v_{l\alpha} \\ v_{l\beta} \\ v_{lo} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} v_{la} \\ v_{lb} \\ v_{lc} \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} i_{l\alpha} \\ i_{l\beta} \\ i_{lo} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_{la} \\ i_{lb} \\ i_{lc} \end{bmatrix} \quad (8)$$

The transformation of simple connection point voltages and line currents of the load allows having the following two equations:

[7,8] Proposes a new reference called $\alpha'\beta'o'$ -axes turning around the axis o of a θ_1 angle to align the instantaneous spatial voltage vector \vec{v}_{pqr} to the p-axis of pqr-axis of reference as specified in Figure 4:

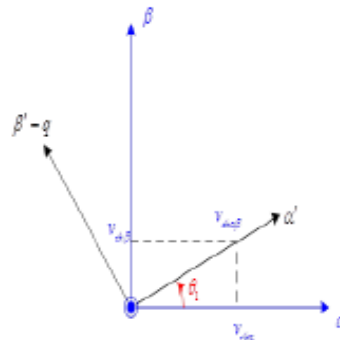


Figure 3. Relation between $\alpha\beta o$ -axes and $\alpha'\beta'o'$ -axes

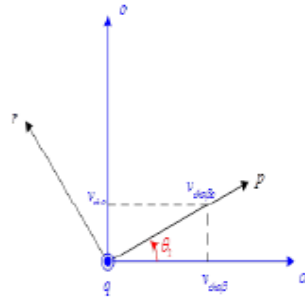


Figure 4. Relation between $\alpha'\beta'o'$ -axes and pqr -axes

This transformation has for currents:

$$\begin{bmatrix} i_{\alpha}' \\ i_{\beta}' \\ i_{o}' \end{bmatrix} = \begin{bmatrix} \cos \theta_1 & \sin \theta_1 & 0 \\ -\sin \theta_1 & \cos \theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{\alpha} \\ i_{\beta} \\ i_{o} \end{bmatrix} \quad (9)$$

In the same way, we find the following relations:

$$\cos \theta_1 = \frac{v_{cha}}{v_{chah}}, \sin \theta_1 = \frac{v_{chb}}{v_{chah}} \text{ and } v_{chah} = \sqrt{v_{cha}^2 + v_{chb}^2} \quad (10)$$

Then [7] proposed to make another transformation of the $\alpha'\beta'o'$ -axes 'by a rotation of the axes around the axis β' , what was known as the pqr -axes as shown in Figure 3.

Then we get:

$$\begin{bmatrix} i_p \\ i_q \\ i_r \end{bmatrix} = \begin{bmatrix} \cos \theta_2 & 0 & \sin \theta_2 \\ 0 & 1 & 0 \\ -\sin \theta_2 & 0 & \cos \theta_2 \end{bmatrix} \begin{bmatrix} i_{\alpha}' \\ i_{\beta}' \\ i_{o}' \end{bmatrix} \quad (11)$$

In the same way, we find:

$$\cos \theta_2 = \frac{v_{l\alpha}}{v_{l\alpha\beta o}}, \sin \theta_2 = \frac{v_{l\beta}}{v_{l\alpha\beta o}}, v_{l\alpha\beta} = \sqrt{v_{l\alpha}^2 + v_{l\beta}^2} \text{ and } v_{l\alpha\beta o} = \sqrt{v_{l\alpha}^2 + v_{l\beta}^2 + v_{l\beta o}^2} \quad (12)$$

After a simple development, we will have the final relationship between the currents in the pqr -axes and $\alpha\beta o$ -axes:

$$\begin{bmatrix} i_p \\ i_q \\ i_r \end{bmatrix} = \begin{bmatrix} \frac{v_{l\alpha}}{v_{l\alpha\beta o}} & \frac{v_{l\beta}}{v_{l\alpha\beta o}} & \frac{v_{l\beta o}}{v_{l\alpha\beta}} \\ -\frac{v_{l\beta}}{v_{l\alpha\beta}} & \frac{v_{l\alpha}}{v_{l\alpha\beta}} & 0 \\ -\frac{v_{l\alpha} v_{l\beta o}}{v_{l\alpha\beta o} v_{l\alpha\beta}} & -\frac{v_{l\beta} v_{l\beta o}}{v_{l\alpha\beta o} v_{l\alpha\beta}} & \frac{v_{l\alpha\beta}}{v_{l\alpha\beta o}} \end{bmatrix} \begin{bmatrix} i_{\alpha} \\ i_{\beta} \\ i_{o} \end{bmatrix} \quad (13)$$

The expression of the instantaneous active and reactive power in the pqr -axes is given by:

$$\begin{bmatrix} p_{lp} \\ p_{lq} \\ q_{lr} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} i_{lp} \\ i_{lq} \\ i_{lr} \end{bmatrix} \quad (14)$$

Table 2 summarizes the modes of possible compensations:

Table 2. Modes of compensation of the *pqr* theory

Type of compensation	Parameter of control
Compensation of the harmonic of currents	$p_f^* = \tilde{p}_l, q_{fr}^* = \tilde{q}_{lr}$ et $q_{fq}^* = \tilde{q}_{lq}$
Compensation of the harmonic of currents and reactive power	$p_f^* = \tilde{p}_l, q_{fr}^* = q_{lr}$ et $q_{fq}^* = q_{lq}$

By compensating for the harmonics of current and reactive power the currents of reference of the filter in the PQR reference are calculated from the equation (14) as follows:

$$\begin{bmatrix} i_{lp} \\ i_{lq} \\ i_{lr} \end{bmatrix} = \frac{1}{v_{l\alpha\beta o}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} p_{lp} \\ p_{lq} \\ q_{lr} \end{bmatrix} \quad (15)$$

Figure 5 presents schematically the principal of the PQR theory.

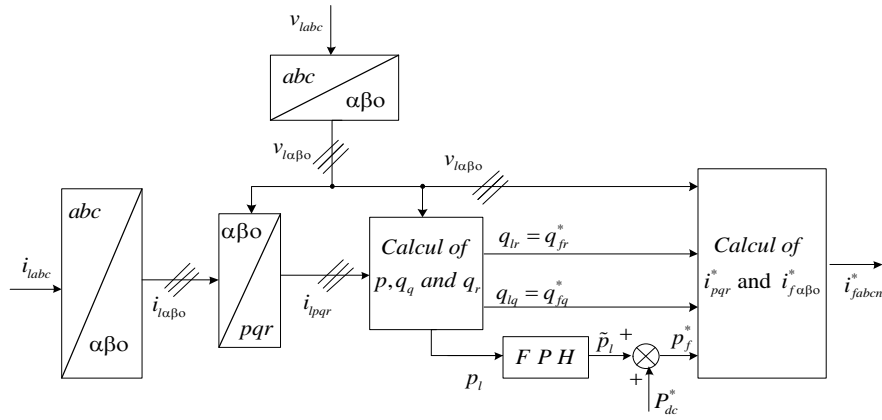


Figure 5. Schematic diagram of the *pqr* theory

The system parameters considered for simulation and the load specifications are given in table 4.

Table 4. System parameters for simulation and load specifications

Capacitance of the capacitor	5 mF
coupling impedance R_f, L_f	0.1 m Ω , 0.1 mH
the source voltage and frequency	220 V, 50Hz
Source impedance R_s, L_s	1 m Ω , 1 mH
Line impedance R_{ch}, L_{ch}	1 m Ω , 1 mH
Load impedance R_l, L_l	5 Ω , 10 mH

Results and Discussion

Simulation results for the proposed control strategies are shown in Figures 6 to 15.

Simulation results before compensation

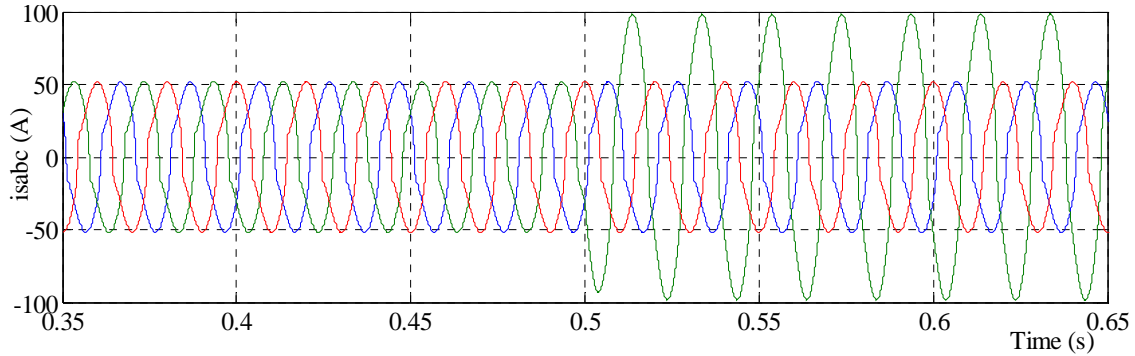


Figure 6. Phase Currents before compensation

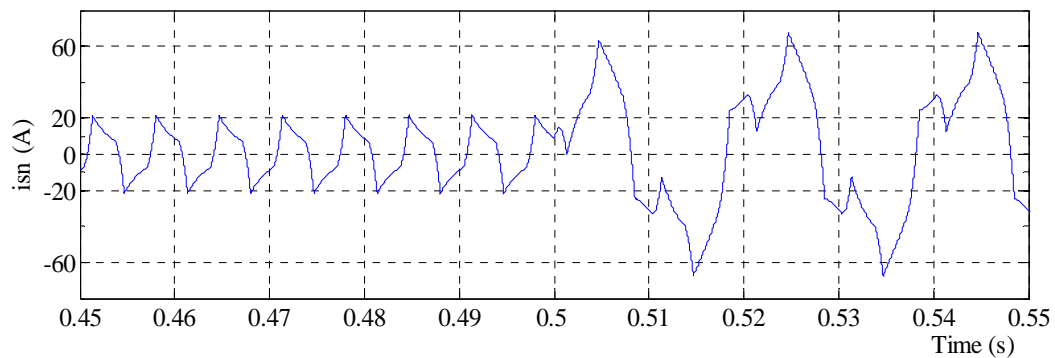


Figure 7. Neutral Current before compensation

Figure 6 shows the current load waveform. This is a very distorted and non-sinusoidal before and after unbalanced.

Figure 7 shows the shape of the neutral current with a maximum value of 20A before unbalanced load and 60A after in the unbalanced load.

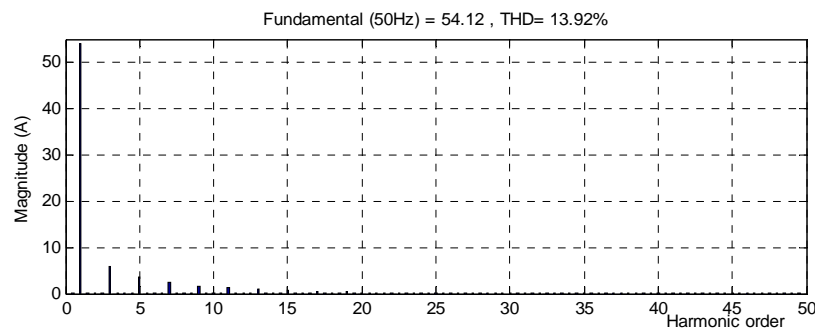


Figure 8. %THD load current before compensation

Simulation results with the CV theory

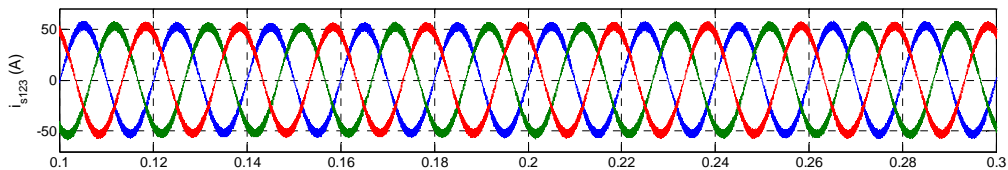


Figure 9. Source currents after compensation with the CV theory

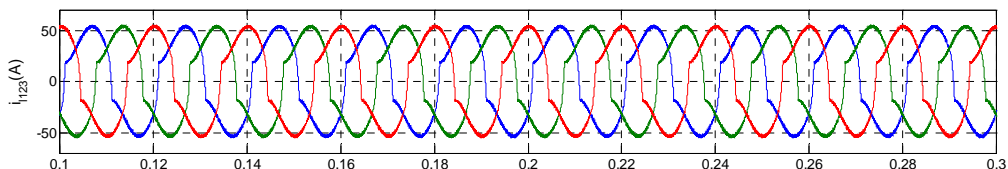


Figure 10. Load current after compensation with the CV theory

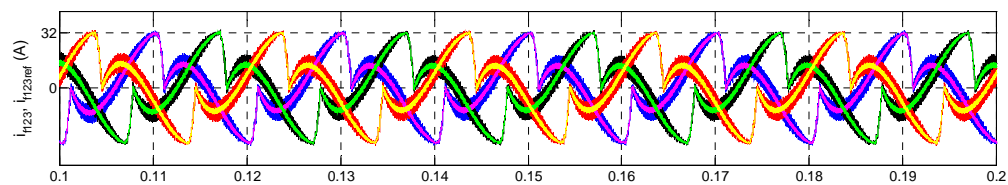


Figure 12. Injected currents by the active power filter with the CV theory

Simulation results with the PQR theory

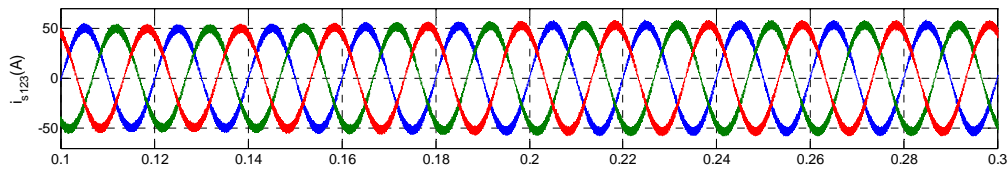


Figure 13. Source currents after compensation with the pqr theory

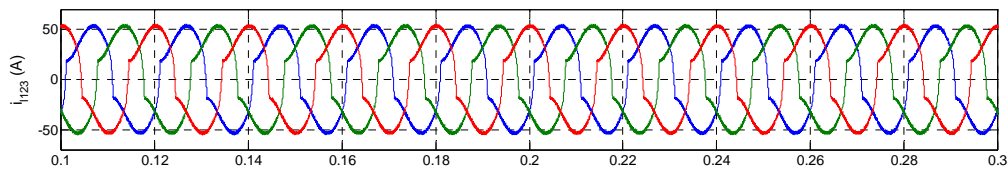


Figure 14. Load current after compensation with the pqr theory

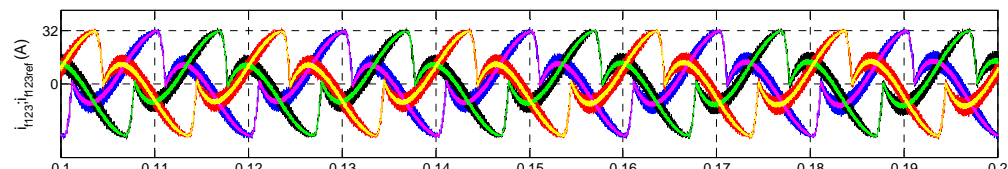


Figure 15. Injected currents by the active power filter with the pqr theory

Figures 9 shows the source current waveform after filtering which is sinusoidal.

Figures 10 and 14 represent the shape of the load current, it is a non-sinusoidal current and highly deformed. Figures 11 and 15 show the shape of the neutral current which does not exceed its maximum value 11.5 A. Figures 12 and 16 represents the active power filter injected currents.

Figures 13 shows the first phase source current's THD for the two theories, the total harmonic distortion (THD) is 1.67% for the CV theory Figure 13 and 1.54% for the PQR theory.

Conclusion

From the obtained results it is seen that the compensation performance of APF is almost similar with all two methods, with the advantage of avoiding the time-consuming $\alpha\beta$ -axes to PQR-axes.

Also, the controller was capable of eliminating the current flowing in the neutral line. Finally, the simulation results validated both the steady state and dynamic behavior of the proposed controller.

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