Expert system for detecting and diagnosing car engine starter cranks fault using dynamic control system

David Ibitayo LANLEG*, Umar Muhammad GARBA, Usman Mohammed GANA, Abdulrahman ABDULGANIYU

Department of Mathematics/Computer Science, Ibrahim Badamasi Babangida University Lapai-Niger State, Nigeria
Email: loislanlege@yahoo.com

* Corresponding author, phone: +2348030528667, 08156073449

Abstract

Application of Dynamic Control Systems (DCS) in detecting and diagnosing car engine Starter Cranks is continuously being implemented to serve different cases of real life problems such as Control of MEMS-based scanning-probe data-storage devices, track-follow control for tape storage, probe-based ultrahigh-density storage technology, a review of feed forward control approaches in nanopositioning for high-speed SPM and so on. Car engine Starter Cranks faults can be detected by sequence of diagnostic processes which brings about the deployment of an Expert System. An Expert System is one of the leading Artificial Intelligence techniques that have been adopted to handle such task. This paper presents the imperatives for an Expert System in developing Dynamic Control Systems for detecting and diagnosing car engine Starter Crank faults through input and output requirements of constructing successful Knowledge-Based Systems. Furthermore, diagnosis of car engine Starter Cranks faults requires high technical skills and experience. thus, DCS provides input and output equations in form of Matrix/Vector State Space Representation (MSSR) which is useful in assisting mechanics for car engine Starter Cranks fault detection and diagnosis via DCS and mathematical Differential Equations (DE’s).
Keywords
Starter Cranks; Differential Equations; Matrix/Vector State Space Representation (MSSR); Dynamic Control Systems (DCS); Expert System; Generating Function; Equations (input, output, state)

Introduction

The definition of an ES (expert system) as proposed by [1] is: “an intelligent computer program that uses knowledge and inference procedures to solve problems that are difficult enough to require significant human expertise for their solution”. The ES is a knowledge-based system that consists of two main modules: the knowledge base and the inference engine. It usually has a knowledge acquisition module and an explanation module as extra components. Systems that utilize the knowledge base approach are more straightforward than the conventional approach. Knowledge is represented explicitly in the knowledge base so that it can be altered with relative ease. This representation often takes the form of rules. The inference engine utilizes the knowledge base contents to solve a particular problem according to the user responses through an interface (e.g. enter the symptoms of the car fault). ES provide powerful and flexible means for obtaining solutions to a variety of problems that often cannot be dealt with by other, more traditional and orthodox methods. The terms expert system and knowledge-based system (KBS) are often used synonymously. The four main components of KBS are: a knowledge base, an inference engine, a knowledge engineering tool, and a specific user interface. Some of KBS important applications include the following: medical treatment, engineering failure analysis, decision support, knowledge representation, climate forecasting, decision making and learning, and chemical process control [1,2]. An expert system may completely fulfil a function that normally requires human expertise, or it may play the role of assistance to a human decision maker. The decision maker may be an expert in his own right, in which case the program may justify its existence by improving his productivity [3]. Knowledge acquisition is the transfer and transformation of potential problem-solving expertise from some knowledge source to a program. Knowledge representation is a substantial subfield in its own right, which shares many concerns with both formal philosophy and cognitive psychology. It is concerned with the ways in which
information might be stored and associated in the human brain, usually from a logical, rather than a biological, perspective. An expert system can be distinguished from a more conventional application program in such a way that it stimulates human reasoning about a problem domain, rather than simulating the domain itself. It performs reasoning over representations of human knowledge, in addition to doing numerical calculations or data retrieval. The knowledge base is structured as a set of objects, with rules relating them to corresponding attributes. An object is the conclusion that is defined by its associated rules, while an attribute is a specific quality that with its rule helps define the object. The rule that is applied to an attribute states that an object either has or has not that attribute. These rules occur in sequences and are expression of the form [3]:

If < conditions >, then < actions > If the conditions are true then, the actions are executed.

Or: while values for attributes remain to be input / read value and assign to attribute / evaluate conditions / fire rules whose conditions are satisfied.

When rules are examined by the inference engine, actions are executed if the information supplied by the user satisfies the conditions in the rules. Conditions are expressions involving attribute and logical connective ‘and’. The rule-based expert systems have a wide range of applications for diagnostic tasks where expertise and experience are available but deep understanding of the physical properties of the system is either unavailable or too costly to obtain. In the rule-based systems, knowledge is represented in the form of production rules [3].

Inference engine

In order to execute a rule-based expert system using the method of forward chaining we merely need to fire (or execute) actions whenever they appear on the action list of a rule whose conditions are true [2,4]. This involves assigning values to attributes, evaluating conditions, and checking to see if all of the conditions in a rule are satisfied. A general algorithm for this is highlighted below:

while values for attributes remain to be input
read value and assign to attribute
evaluate conditions
fire rules whose conditions are satisfied

Several points about this require consideration. First, some conflict resolution strategy
needs to be employed in order to decide which rules are fired first. Our method is to fire the rule which the system designer defined first. Also, we wish to cut down on computational time. To do this we must not do anything which does not absolutely need to be done. This means that conditions are only evaluated at the time they might change and that rules are checked (to see if all of their conditions are satisfied) only when they might be ready to be fired, not before. We shall do this as attributes are assigned values and shall only consider rules and conditions affected by the new attribute assignment [4].

**Working memory**

Specific information on a current problem is represented as case facts and entered in the expert system's working memory. The working memory contains both the facts entered by the user from questions asked by the expert system, and facts inferred by the system. The working memory could also acquire information from databases, spreadsheets, or sensors, and be used by the expert system to conclude additional information about the problem by using the general knowledge contained in the knowledge base [4].

**Explanation facility**

Besides providing final results or conclusions, both human experts and expert systems can explain how they arrived at their results. This capability is often important because the types of problems to which expert systems are applied require that a justification of the results be provided to the user. For example, an expert system which recommends some antibiotic treatment for a patient would need to explain to the physician how this recommendation was formulated [4]. Expert systems also have the capability of explaining why a given question is being asked. When an individual consults with a human expert, the conversation is highly interactive, and on occasion, the individual may ask why a certain line of reasoning is being pursued. The explanation given can make the user feel more comfortable with the line of questioning and also help to clarify what issues the expert believes are important for the problem [4].

**User interface**

The user interface is the means of communication between a user and the expert systems problem-solving processes. A good expert system should have an efficient interface.
The user interface in this system will be able to accept the instructions in a form that the user enters and translate them into working instructions for rest of the system. The user will be asked several questions which he would have to answer, subsequently a solution will be displayed thereby. Careful attention should be given to the screen design in order to make the expert system appear friendly to the user [5].

**Expert system for diagnosing the failure of different machines**

One of the earlier published references on diagnostic ES for technical fault diagnosis was developed at MIT in the early 1970’s as stated by Scherer and White in 1989 [6]. Some researches that utilize the expert system for the purpose of diagnosing the failure of different machines are reviewed in the following paragraphs below: Kadarsah proposed and designed a decision model for car fault diagnosis in which an ES was utilized to help inexperienced mechanics and drivers [6]. The model consists of inference engine, knowledge base, database, user interaction and adaptive mechanism. The Inference engine uses backward chaining as a result of a small number of outputs with many possible inputs. In addition, the adaptive mechanism was utilized in the user interaction section in order to receive feedback about system diagnosis result. The feedback results were stored in a database. The adaptive system then processes the stored data and extracts additional rules with the goal of improving the knowledge base. In this system, car faults are divided into three states: Start-up state, Run-stable state and Movement-state. Shell Rule based expert system (CLIPS) with forward chaining inference engine is used in the implementation. CLIPS store’s the knowledge in rules form, which has logic-based representation as well as the production rules. The system interacts with the user through an interface and gives the diagnosis result with illustration. The rule-based expert system contains 150 rules for car failure causes. However, improvement in the domain knowledge and applying adaptive technique for knowledge creation are required in such system [6]. In the work of Peter Nabende and Tom Wanyama, Heavy Duty Diesel Engines (HDDEs) diagnosis was proposed. HDDEs maintenance requires high technical skills and extensive experienced mechanics which are scarce. As a result, employing an expert system in such domain can be highly useful. The HDDE faults diagnosis ES was able to successfully detect malfunctions in the engines and give recommendation of corrective actions. System development leads to collection of valuable information related to HDDE fault diagnosis and training. However, updating the knowledge base affects the
reasoning process performance especially in the continuous run [6]. A research was done by Jindal et al. to assist in the design of an ES for car failure diagnosis and repair [6]. Many factors were considered in this research such as the required time, the place and human expertise level. In addition, the ES development was accompanied by reviewing the technologies used in designing such systems to achieve the best means to be followed. However, the proposed prototype was not promoted to be used as a complete application due to time and resources limitations. Thus, adopting new rules to be performed was an example of further enhancements that the system needed. A survey was done by Milanović et al. for developing moto cultivator fault diagnosis model. This model is based on the hybridization of Expert System (ES) and Decision Support System (DSS) in which ES outcome represents the input to the DSS. The supplier selection for faulty component replacement is made by DSS based on ES outcome. In practice, the designed hybrid system was applied in a small moto cultivator importer and distributor company for servicing purposes. It has proved to be a very useful tool for equipment servicing needs with low development cost. It increases the efficiency of labour and workers’ satisfaction [6].

Conventional programs versus expert systems

According to Durkin [4,7], it is important to understand and appreciate the differences between conventional computer programs and knowledge processing or expert systems. Knowledge processing represents an evolution, rather than a revolution, in the way individuals and computers interact to solve problems. The most basic difference between the two is that conventional programs process data, while expert systems process knowledge. This basic difference influences both the nature of the processing technique used and the results obtained. Conventional programs process data which is usually in numeric form, while an expert system works with symbolic information. Data are isolated bits of information about a problem, whereas symbolic information represents statements or facts concerning the problem which can be used with general knowledge to infer new information. Conventional programs process data by means of algorithms, whereas an expert system will use heuristic reasoning techniques. An algorithm represents a finite set of well-defined steps to be performed. Heuristic reasoning works with the available information to draw conclusions about the problem, but does not follow a prescribed sequence of steps. A conventional program requires complete and precise information. An expert system can work with the
available information whether it is incomplete or uncertain. In this sense, an expert system can provide some results even under the constraints of limited or uncertain information. A conventional program would be severely limited under such constraints. The interface of an expert system permits questions to be asked and answers given using a natural language style. This interface is more readily accepted by end-users than the command interface found with most conventional programs. Interaction with an expert system also follows more closely the conversation between one human obtaining advices from another human. During the conversation, explanations are provided by the expert to queries as to "why" a question is being asked, and "how" a given conclusion was reached. This point makes an expert system considerably different from a conventional program, which simply provides a final answer. Conventional programs provide a final solution usually in the form of a result from a computation. The computation may have involved a complex series of tasks, but the user will only see the final result and not the intermediate steps that led to the final result. Expert systems provide a result in the form of a recommendation, with a justification in the form of a tracing of its reasoning. Given the correct information, conventional programs will provide an exact solution to a problem. It is an "all or nothing" situation. Expert systems can make mistakes, just as a human expert might. This point appears to give the conventional program an advantage over the expert system. However, this appearance is only an illusion. Expert systems work on types of problems which are less structured than conventional programs, and the information available may not be sufficient to obtain an exact solution. However, the expert system will still be able to reach some reasonable conclusion, even if it is not optimal, whereas a conventional program will fail if not provided with all of the information it needs. This ability of an expert system to be able to make decisions in the absence of complete or certain information is the result of developments in the area of inexact reasoning.

**Why use an expert system?**

Like any project venture, developing an expert system must have some justification [4,5]. Insight for justifying an expert system can be gained when one compares an expert system with a human expert. One can formulate several general reasons for employing an expert system such as: replacement of human expert, assistant to human expert, transfer of expertise to novice [4]. Using an expert system to replace a human expert is done primarily to use the system when the expert is not available. For example, through time constraints, the
human expert may not be available, while an expert system designed to control some manufacturing process would be available 24 hours a day. Another expert system, containing the expertise of a unique expert within a company, could be made available to company sites located in other geographic areas. If the expert should leave or retire from the company, the expertise captured in the expert system could serve as a replacement for the expert. Human experts may be scarce, hence expensive. Expert systems, by contrast, may be inexpensive. Developing an expert system can be a costly venture, but the finished product would have low operating costs. The finished system can also be duplicated at low cost and distributed widely [4,5]. In the area of science, justifying an expert system for replacing a human can be found in such applications as space exploration [4], or providing the expertise of a geophysicist to some remote oil exploration site [4]. Another example would be to replace the human operator of a control process. Assisting a human expert is one of the most commonly found applications of expert systems. In this application, the expert system attempts to aid the human expert in a routine or mundane task. For example, a physician may have general knowledge of most diseases, but could use some additional support in diagnosing a given problem with a patient. In another example, a bank manager may be responsible for processing numerous loan applications, but could use help with some of the routine decisions made. In both applications, the human expert is fully capable of performing the task, but obtains additional support from the expert system. In this type of expert system application, the objective is to improve the overall productivity of the current practice. One specialized application of an expert system which can be used to assist the expert is the ability of the expert system to learn about a specific problem. The most common learning method used in expert systems today is a technique known as induction [4,7]. The induction technique works with information contained in a set of examples to induce a set of rules which capture the knowledge about the problem. This approach has particular value for those problems where the expert lacks the knowledge to form decisions, but has a history of data on the problem. The induction technique can uncover classifications in the data which can be used for guiding the decision process. The expertise held by a human expert is a valuable resource. Knowledge is gained by the expert through years of experience from working on the problem. In many organizations, it is important that this expertise not be lost, but transferred to others through training. An expert system can be developed to accomplish this training task.
Material and method

Dynamics Control System (DCS)

In applied mathematics and engineering the central theory deals with the behaviour of dynamical system over time. The dynamic behaviour of a system may therefore be understood by studying their mathematical description. For instance, the flight path of an airplane subject to certain engine thrust, rudder elevation angles and particular wind condition or the current flowing in an electrical circuit consisting of interconnections of resistors, inductors, capacitors, transistors, diodes, voltage or current source etc can be predicted using mathematical description of the pertinent behaviour. Mathematical equations in the form of Differential or difference equations are used to describe the behaviour of the process usually referred to governing equations whose solutions give the required response of the particular system under consideration.

A system is a group of component part put together to accomplish a certain task. It is also said to be an arrangement or collection of things connected or related in such a manner as to form an entire whole. Simply, a system is an arrangement of physical component connected or related in such a manner as to form and or act an entire unit. Whereas the concept of Control is analogous as either to direct, regulate or to command. Thus; a Control System is an arrangement of physical components connected or related in such a manner as to command, direct and regulate itself or another system. It is therefore important to note that a control system is made up of three components namely; input, process and output

Mathematical classification of systems

In this paper we shall not dwell on a comprehensive classification of systems as this may not give the much desired understanding of the concept. Hence an enumeration of the more common classes of systems most often encountered in field of engineering and science is of high consideration. Any particular set of equation describing a given system generally depends on the effect to be captured. Some of these systems may include Lumped Parameter or Finite-Dimensional Systems; Distributed Parameters or infinite – Dimensional Systems; Continuous- Time and discrete – Time Systems; Deterministic and Stochastic Systems and appropriate combination of any of the fore mentioned is known as hybrid systems. It must however be noted that the appropriate mathematical setting for Finite–Dimensional System are Finite-Dimensional Vector Spaces and for infinite–Dimensional system are defined
Infinite Dimensional Linear Spaces. Continuous–Time Finite–Dimensional Systems are described by Ordinary Differential Equations or some kinds of integral Equations while Discrete-Time Finite Dimensional Systems are governed by Ordinary Difference equations or Discrete-Time Counterparts to those Integral equations. The governing differential equations to Infinite–Dimensional Systems include partial Differential equations Volterra intergro-Differential Equations, Functional Equations etc.

**Finite-dimensional system**

This is mainly concerned with continuous- time and Discrete-time finite dimensional system. The continuous–time finite dimensional dynamic system for our consideration will be those described by the following set of governing differential equations:

\[
\begin{align*}
\dot{x}_i &= f_i(t, x_1, x_2, \ldots, x_n, u_1, u_2, \ldots, u_m), \ i=1,2,3,\ldots,n \\
\dot{y}_i &= g_i(t, x_1, x_2, \ldots, x_n, u_1, u_2, \ldots, u_m), \ i=1,2,3,\ldots,p
\end{align*}
\]

(1a)

where \(u_i, i=1(1)m\) denote the inputs or the stimuli; \(y_i, i=1(1)p\) denote the outputs or responses; \(y_i, i=1(1)n\) represent state variable, \(x_i, i=1(1)n\) represents state variables; \(t\) denotes time, \(\dot{x}_i\) denotes time derivates of state variables; \(f_i, i=1,2,3\ldots,n\) real value functions of \(1+m+n\) variables; \(g_i, i=1,2,3\ldots,p\) real value functions of \(1+m+n\) real variables.

A complete description of the system will usually require a set of initial conditions;

\[
X_i(t_0)=x_i(0), \ i=1,2,3,\ldots,n
\]

(1b)

where \(t_0\) is initial time.

In most cases of practical application, there often arise the need to impose constraints on the quantities \(f_i, g_i\) and \(u_i\).

**Opened-loop and closed-loop control system**

Controlled system is classified into two categories: Opened–loop and Closed–loop. An opened–loop controlled system is that in which the control action is independent of the output, while a closed–loop controlled system is that in which the control action is somehow dependent on the output. As such the system of car engine overheating is categorized under the closed loop control system due to the fact that respective inputs depend upon the respective outputs of the system. Feedback is also a characteristic of a closed–loop controlled system. This is defined as that property of the closed loop control system which permit the output (or some other control variable) to be compared with the input to the system (or an output to other internally situated components or subsystem) so that the appropriate action
may be formed as some function of the output and input. Some of the characteristics of feedback include increases accuracy oscillation, tendency towards instability, reduces the sensitivity of ratio output to variation in system parameters, reduces effect of nonlinearity, reduces the effect of external disturbance or noise, increases bandwidth.

For a given set of differential equations describing a certain dynamic system (system that changes with time) there is need to know the set of state variables and there number in the system. That is: How many state variables are involved? What are these state variables?

For the first, the number of state variables is equal to the total number of initial conditions required to completely solve the differential equations of overheating for a car engine. For instance, if a dynamic system is described by a single second order differential equation then two initial conditions is required to completely solve the differential equation. Thus there are two state variables for this system. For (ii); these variables for which initial conditions are required for the solution of the governing differential equation defined above are chosen as the required state variable

**General formulation**

Once the state variables are appropriately selected the next step is to construct the state variable equations. These state equations are system of first order differential equations in the state variables on the left hand side and algebraic system (function) of the state variables as system input and possibly time on the right hand side. In general for a multi- input multi-output system with m state variables we have

\[ x_1, x_2, x_3, \ldots, x_m \] inputs \( u_1, u_2, u_3 \ldots u_p \) and \( r \) outputs.

\[ y_1, y_2, y_3, \ldots, y_r \] the state variable equations are given in this form:

\[
\begin{align*}
\dot{x}_1 &= f_1(x_1, x_2, \ldots, x_m, u_1, u_2, \ldots u_p, t) \\
\dot{x}_2 &= f_2(x_1, x_2, \ldots, x_m, u_1, u_2, \ldots u_p, t) \\
\dot{x}_3 &= f_3(x_1, x_2, \ldots, x_m, u_1, u_2, \ldots u_p, t) \\
&\vdots \\
\dot{x}_m &= f_m(x_1, x_2, \ldots, x_m, u_1, u_2, \ldots u_p, t)
\end{align*}
\]

where \( f_i \) are in general non linear functions of the arguments.

Similarly, the system output variables may also be expressed as follows:
\[ \dot{y}_1 = g_1(x_1, x_2, \ldots, x_m, u_1, u_2, \ldots, u_p, t) \]
\[ \dot{y}_2 = g_2(x_1, x_2, \ldots, x_m, u_1, u_2, \ldots, u_p, t) \]
\[ \dot{y}_3 = g_3(x_1, x_2, \ldots, x_m, u_1, u_2, \ldots, u_p, t) \]

where \( g_k \) are in general non-linear functions.

In the event that non-linear elements are present in the system the functions \( f_j(j=1(1)m) \) and \( g_k(k=1(1)m) \) also turn out to be non-linear and quite complex in nature thereby making the analysis or solution complicated.

**Matrix vector formalism**

This involves the representation of equations (1-2) more conveniently using matrix-vector form by the following definitions:

\[ X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} \text{ is state vector, } \quad F = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_m \end{bmatrix} \text{ as the function, } \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} \text{ output vector, } \]

\[ G = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_m \end{bmatrix} \text{ algebraic Function and } \quad U = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix} \text{ input vector } \]

Thus, the state variable equations are represented by:

\[ \dot{x} = f(x, u, t) \]  

(4)

And the system output as:

\[ \dot{y} = g(x, u, t) \]

(5)

As expected the complexity associated with the general formulation reduces considerably for the case of a linear system. In the event that all the elements in the model of a dynamical system are linear the algebraic functions \((f_j \text{ and } g_j)\) appearing in equations (7) and (7) will take the following special forms:

\[ \begin{align*}
\dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + \cdots + a_{13}x_3 + b_{11}u_1 \cdots + b_{1r}u_r \\
\dot{x}_2 &= a_{21}x_1 + a_{22}x_2 + \cdots + a_{23}x_3 + b_{21}u_1 \cdots + b_{2r}u_r \\
\dot{x}_3 &= a_{31}x_1 + a_{32}x_2 + \cdots + a_{33}x_3 + b_{31}u_1 \cdots + b_{3r}u_r \\
& \vdots \\
\dot{x}_{m-1} &= a_{m-1,1}x_1 + a_{m-1,2}x_2 + \cdots + a_{m-1,m}x_m + b_{m-1,1}u_1 \cdots + b_{m-1,r}u_r \\
\dot{x}_m &= a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_m + b_{m1}u_1 \cdots + b_{mr}u_r
\end{align*} \]

(6)
\[ y_1 = c_{11}x_1 + c_{12}x_2 + \cdots + c_{1m}x_m + d_{11}u_1 + d_{12}u_2 + \cdots + d_{1p}u_p \]
\[ y_2 = c_{21}x_1 + c_{22}x_2 + \cdots + c_{2m}x_m + d_{21}u_1 + d_{22}u_2 + \cdots + d_{2p}u_p \]
\[ y_1 = c_{31}x_1 + c_{32}x_2 + \cdots + c_{3m}x_m + d_{31}u_1 + d_{32}u_2 + \cdots + d_{3p}u_p \]
\[ \vdots \]
\[ y_{r-1} = c_{r-1,1}x_1 + c_{r-1,2}x_2 + \cdots + c_{r-1,m}x_m + d_{r-1,1}u_1 + d_{r-1,2}u_2 + \cdots + d_{r-1,p}u_p \]
\[ y_r = c_{r1}x_1 + c_{r2}x_2 + \cdots + c_{rm}x_m + d_{r1}u_1 + d_{r2}u_2 + \cdots + d_{rp}u_p \]

By defining the following quantities to enable us represent the formulations above in matrix-vector form:

\[
A = \begin{bmatrix}
    a_{11} & a_{12} & \cdots & a_{1m} \\
    a_{21} & a_{22} & \cdots & a_{2m} \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{m1} & a_{m2} & \cdots & a_{mm}
\end{bmatrix}_{m \times m} = \text{State matrix} \quad B = \begin{bmatrix}
    b_{11} & b_{12} & \cdots & b_{1p} \\
    b_{21} & b_{22} & \cdots & b_{2p} \\
    \vdots & \vdots & \ddots & \vdots \\
    b_{m1} & b_{m2} & \cdots & b_{mp}
\end{bmatrix}_{m \times p} = \text{Input matrix}
\]

\[
C = \begin{bmatrix}
    c_{11} & c_{12} & \cdots & c_{1m} \\
    c_{21} & c_{22} & \cdots & c_{2m} \\
    \vdots & \vdots & \ddots & \vdots \\
    c_{r1} & c_{r2} & \cdots & c_{rm}
\end{bmatrix}_{r \times m} = \text{Output Matrix} \quad D = \begin{bmatrix}
    d_{11} & d_{12} & \cdots & d_{1m} \\
    d_{21} & d_{22} & \cdots & d_{2m} \\
    \vdots & \vdots & \ddots & \vdots \\
    d_{r1} & d_{r2} & \cdots & d_{rp}
\end{bmatrix}_{r \times p}
\]

Therefore, the final form of the system in the matrix–vector form is given as:

\[ \dot{X} = AX + BU \quad \text{State Equation} \quad (10) \]
\[ \dot{Y} = CX + DU \quad \text{Output Equation} \quad (11) \]

The above system of equation is known as the state-space representation or state–space form of the system model. This very convenient form of representing a system model is particularly useful in the analysis and control of a dynamic system.

**Governing differential equations for diagnosing starter cranks**

Suppose when starting there is said to be Stalls then the DDE is given by:

\[ \frac{1}{\gamma_1} (\dddot{x}_i + \dddot{x}_j + \dddot{x}_l + \dddot{x}_4) = f_1 \quad (13a) \]

But if there is no fire in the engine the DDE is thus given as:

\[ \frac{1}{\gamma_2} (\dddot{x}_2 + \dddot{x}_2 + \dddot{x}_2 + \dddot{x}_2) = f_2 \quad (13b) \]

Also whenever there is said to be no spark from the plug the governing DDE is
therefore formulated as below:

$$\frac{1}{\gamma_3} (\ddot{x}_3 + \dot{x}_3 + x_3) = f_3$$  \hspace{1cm} (14)$$

$$\frac{1}{\gamma_4} (\ddot{x}_4 + \dot{x}_4 + x_4) = f_4$$  \hspace{1cm} (15)$$

**Figure 1**: Expert/Dynamic System Representation Using Block Diagram (where $\sigma$ and $\varphi$ represent responses of sequences of decisions to be responded as either (Yes/No) respectively while $\omega$ represents the total chances of decisive events that requires an absolute response at a time, $f_n$ is the input generating function and $\gamma_n$ is the equiprobable response generating function)

Similarly, if there is said to be a resulting Spark from Coil then the DDE for the decisive selection is thus formulated as:

$$\frac{1}{\gamma_5} (\ddot{x}_5 + \dot{x}_5 + x_5) = f_5$$  \hspace{1cm} (16)$$

When Stall is in the Rain the governing dynamic DE becomes:

$$\frac{1}{\gamma_6} (\ddot{x}_6 + \dot{x}_6 + x_6) = f_6$$  \hspace{1cm} (17)$$

When starter spins we have:
\[
\frac{1}{\gamma_3}(\ddot{x}_7 + \ddot{x}_1 + \ddot{x}_1 + x_1) = 2f_\gamma + f_8
\]  
Equation (18)

By equating equations (12) and (14):

\[
\frac{1}{\gamma_1}(\ddot{x}_i + \ddot{x}_i + \ddot{x}_i + x_i) = f_i
\]  
Equation (19)

Again by equating (13) and (17):

\[
\frac{1}{\gamma_2}(\ddot{x}_2 + \ddot{x}_2 + \ddot{x}_2 + x_2) = f_2
\]  
Equation (20)

Similarly choosing equations (12) and (18):

\[
\frac{1}{\gamma_1}(\ddot{x}_i + \ddot{x}_i + \ddot{x}_i + x_i) = f_i
\]  
Equation (21)

\[
\frac{1}{\gamma_2}(\ddot{x}_7 + \ddot{x}_7 + \ddot{x}_7 + x_7) = f_7
\]  
Equation (22)

Equations (19) and (20) we thus have the State Variable Equations (SVE) as:

\[
x_1 = y_1, \quad \dot{x}_3 = y_2, \quad \dot{x}_1 = y_3, \quad y_1 = x_1, \quad y_2 = x_3, \quad y_3 = \dot{x}_3, \quad y_4 = \dot{x}_1, \quad y_5 = \ddot{x}_1, \quad y_6 = \dddot{x}_1
\]

\[
y_6 = \dddot{x}_3, y_5 = \dddot{x}_1
\]

\[
\dot{y}_1 = y_3
\]

\[
\dot{y}_2 = y_4
\]

\[
\dot{y}_3 = y_5
\]

\[
\dot{y}_4 = y_6
\]

\[
\dddot{y}_6 = (\gamma_3f_3 - (y_6 + y_4 + y_2)) = \gamma_3f_3 - y_2 - y_4 - y_6
\]

\[
\dddot{y}_5 = f(y_5 + y_3 + y_1) = -(y_1 + y_3 + y_5) + f\frac{1}{\gamma_1} = -y_1 - y_3 - y_5 + \frac{1}{\gamma_1} f_1
\]

Expressing equation (25) in a matrix vector form gives the Matrix State Space Representation (MSSR) of the State Variable Equation (SVE) and hence the State Equation (SE) can be deduced thereafter.

\[
\begin{pmatrix}
\dot{y}_1 \\
\dot{y}_2 \\
\dot{y}_3 \\
\dot{y}_4 \\
\dot{y}_5 \\
\dot{y}_6
\end{pmatrix} =
\begin{pmatrix}
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
-1 & 0 & -1 & 0 & -1 & 0 \\
0 & -1 & 0 & -1 & 0 & -1
\end{pmatrix}
\begin{pmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4 \\
y_5 \\
y_6
\end{pmatrix} +
\frac{1}{\gamma_1}
\begin{pmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4 \\
y_5 \\
y_6
\end{pmatrix}
\begin{pmatrix}
f_1 \\
f_2 \\
f_3
\end{pmatrix}
\]

Equation (26)
Hence the State Equation (SE) for diagnosing start and running process becomes:

\[ \dot{Y} = AZ + BF \]  \hspace{1cm} (27)

Using equation (21) and (22) to formulate another SVE as well as MSSR for the given system for the described system to be decisively diagnosed. Thus; equation (28) below gives SVE for the described system as:

However; from equation (21) and (22) we thus have the State Variable Equations (SVE) formulated as:

\[ \begin{align*}
q_1 &= x_1, q_2 = x_7, q_3 = \dot{x}_1, q_4 = \dot{x}_7, q_5 = \ddot{x}_1, q_6 = \ddot{x}_7, q_7 = \dddot{x}_1 \quad \text{and} \quad \dot{q}_5 &= \dddot{x}_1 \\
\dot{q}_1 &= q_3 \\
\dot{q}_2 &= q_4 \\
\dot{q}_3 &= q_5 \\
\dot{q}_4 &= q_6 \\
\dot{q}_5 &= -q_5 - q_3 - q_1 + \gamma_1 f_1 \\
\dot{q}_6 &= -q_6 - q_4 - q_2 + \gamma_2 f_2
\end{align*} \]  \hspace{1cm} (31)
Again; the State Equation (SE) is given by equation (33) as:

\[ Q = AQ + BF \]

Practical Illustration of an Expert System using Dynamic Control System

Output Equations

From equation (30) suppose the input is say \( f_1 \) (Starter Cranks?) i.e.:

\[
\begin{pmatrix}
\dot{q}_1 \\
\dot{q}_2 \\
\dot{q}_3 \\
\dot{q}_4 \\
\dot{q}_5 \\
\dot{q}_6 \\
\end{pmatrix} =
\begin{pmatrix}
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
-1 & 0 & -1 & 0 & -1 & 0 \\
0 & -1 & 0 & -1 & 0 & -1 \\
\end{pmatrix}
\begin{pmatrix}
q_1 \\
q_2 \\
q_3 \\
q_4 \\
q_5 \\
q_6 \\
\end{pmatrix} +
\begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
\gamma_1 \\
\gamma_7 \\
\end{pmatrix} F(t)
\]

(32)

Figure 2. A Test Interface for the Detection of Starter Cranks

Suppose the response is Yes; then the resulting output equation becomes:

\[
y_2 = z_1 = \alpha \left( \frac{C_1 + D_2 G}{\alpha + \beta} \right) \text{; \hspace{0.5cm} i.e.: } C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}, F = \begin{bmatrix} 0 \\
0 \\
0 \\
\gamma_6 \\
\gamma_7 \\
\end{bmatrix}
\]

(34)

And the equivalence Expert System graphics output is:
**Figure 3. A Test Interface for the Detection of Engine Fires**

Similarly, if the input (response implied) as $f_2$ (Engine Fires?) is Yes (Input Response) then the governing output equation becomes:

$$y_2 = z_2 = \alpha \frac{(CI + DG)}{\alpha + \beta}; \quad ie: \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}, \quad F = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ y_6 \\ y_2 \end{bmatrix}$$

(35)

And the equivalence Expert System graphics output is as shown in Figure 5.
Figure 5. A Test Interface for the Detection of OBD/Blink Code

Again, if the input (response implied) is $f_3$ (Check OBD, blink code?) and input response is Yes from the interface in fig 4 then the governing output equation becomes:

$$y_3 = z_3 = \alpha \frac{(C_1 + D_G)}{\alpha + \beta}; \quad\text{ie:}\ C = [0 \ 0 \ 1 \ 0 \ 0], \ D = [0 \ 0 \ 0 \ 0], \ F = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \gamma_6 \\ \gamma_2 \end{bmatrix}$$

(36)

And the equivalence Expert System graphics output is as shown in Fig 6.

Figure 6. A Test Interface for the Detection of Stalls on Key release

Also, if the input (response implied) is $f_4$ (Stalls on key release to run?) and input response is Yes then the governing output equation becomes:
\[ y_4 = z_4 = \frac{\alpha (CI + DG)}{\alpha + \beta}; \quad \text{ie:} \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}, \quad F = \begin{bmatrix} y_6 \\ y_7 \end{bmatrix} \] (37)

And the equivalence Expert System graphics output is as shown in Fig. 7.

**Figure 7. A Test Interface for the Detection of Circuit/Column key Switch Failure**

Therefore; the suggested output gives the general solution of the problem as shown in fig 7 for an ES and represented by equation (37) for the sequence of responses (Yes) for a DCS. This is to say that the detected problem is ‘ignition “run” Circuit or column key switch failure, Ring out with meter.

**Discussion**

It is very clear that throughout the illustrations of a DCS via an ES that DCS is such a system that simply can sufficiently be used to fully represent the decisive situations of an ES. In addition when considering the DDE formulated especially in equations (34-37) and the equivalence interface representation of an ES which helps in the interpretation of the formulated DDC especially to the world where the mathematical language becomes difficult and complicated to the ordinary World’s interpretation. Also, the DDE’s considered in this paper has established a ground through which every decisive system can be expressed through DCS more effectively and in a logically systematic form.

According to another aspect of the present invention, the present invention provides a
dynamical system for tracking a reference signal, including [9]: a controlled element for being set according to a provided control signal; a controller for feedback controlling the controlled element by providing the control signal, wherein the controller is adapted to feedback control the controlled element depending on a provided reference signal and a feedback signal indicating an actual state of the controlled element [10]; and an impulsive state modification block for modifying an internal state of the controller at one or more discrete time instants, while applying the control function at time periods not including the discrete time instants, wherein the impulsive state modification block is adapted to modify the internal state depending on a signal external to the controller [6,11].

However a DCS has a more logically and a mathematically established system that uses concepts capable of handling any form of ES for a given sequence of decision making especially as the type considered in the detection of Car engine tarter cranks fault. The main objective is to express a dynamically formulated system of decisions requiring responses as the case of a typical ES which has successfully been illustrated Using the concept of DCS. It is then however said that the equation below gives a general stage process of decision making for detecting car engine starter cranks fault by an ES using a modelled DCS output general equation of the form:

\[
y = CE + DU, \forall \text{ given input/output say } h_n y_n = (\alpha \beta)_n \frac{(CE + DH)}{\alpha_n + \beta_n};
\]

\[
ie: C = [n \times n] \text{idemtity matrix}
\]

\[
E = \begin{bmatrix}
E_1 \\
E_2 \\
E_3 \\
E_4 \\
E_5 \\
\vdots \\
E_n
\end{bmatrix}, D = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} \text{ie: } y_n = h_n = E_n, \forall n = 1,2,3,...H = \begin{bmatrix} 0 \\
0 \\
0 \\
0 \\
\eta_1 \\
\eta_2 \end{bmatrix}
\]

Conclusively, for the given illustrations in Figures 2-7 is however important to say that figure 7 gives the detected fault as ‘ignition “run” Circuit or column key switch failure, Ring out with meter’ and the equivalence solution given by equation (37) as obtained by applying Dynamic Control System (DSC) approach.
References